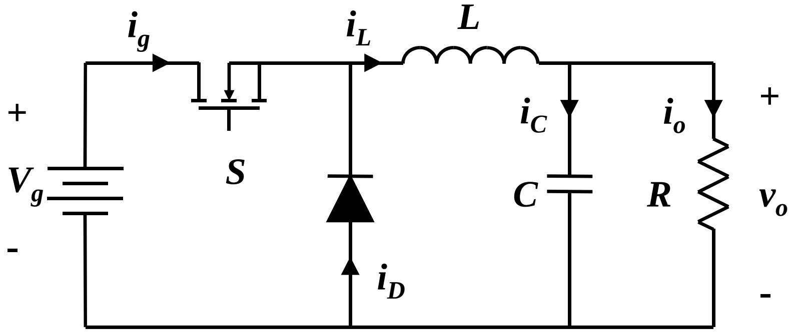
Modeling DC – DC converters in DCM

- Steady-state model
- Dynamic model
- Average paradox
- Reduced order model
- Complete order model
- Continuous model
- Discrete small-signal model
- Simulation
- Conclusions and final remarks
- References

Steady-state model in DCM

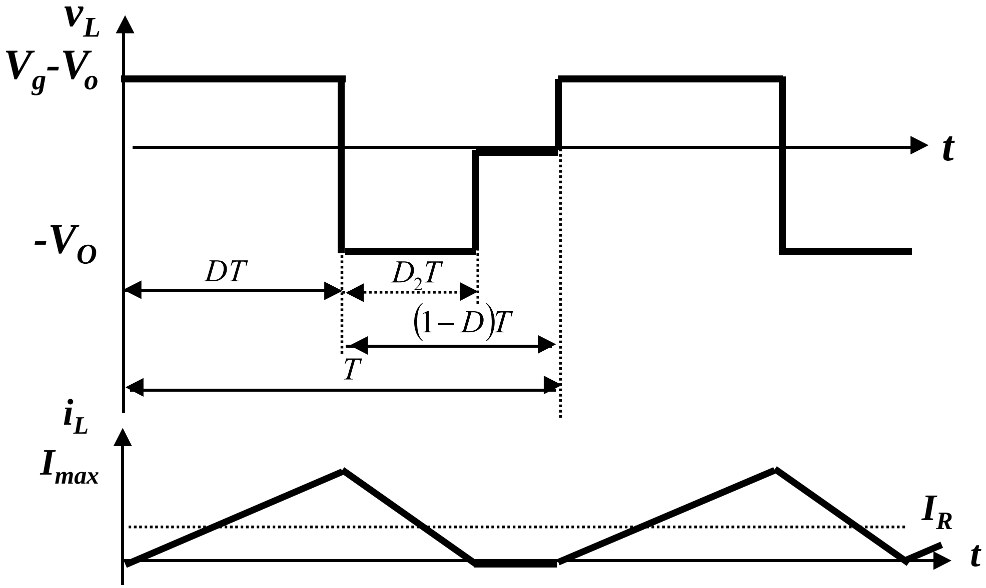
Buck DCM

$$\frac{V_o}{V_g} = \frac{2}{1+\sqrt{1+\frac{4K}{D^2}}}$$



$$I > \Delta i_L \quad for \quad CCM$$

$$I < \Delta i_L \quad for \quad DCM$$

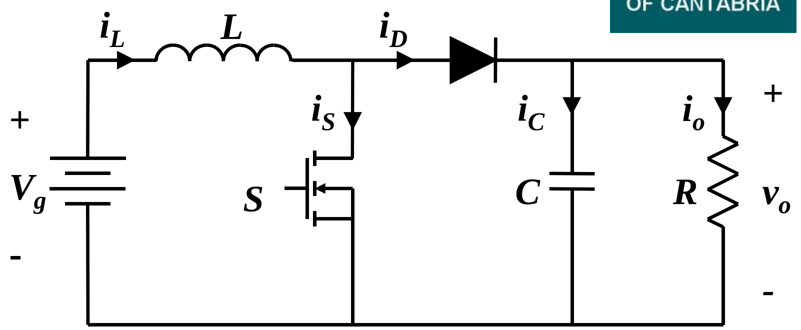


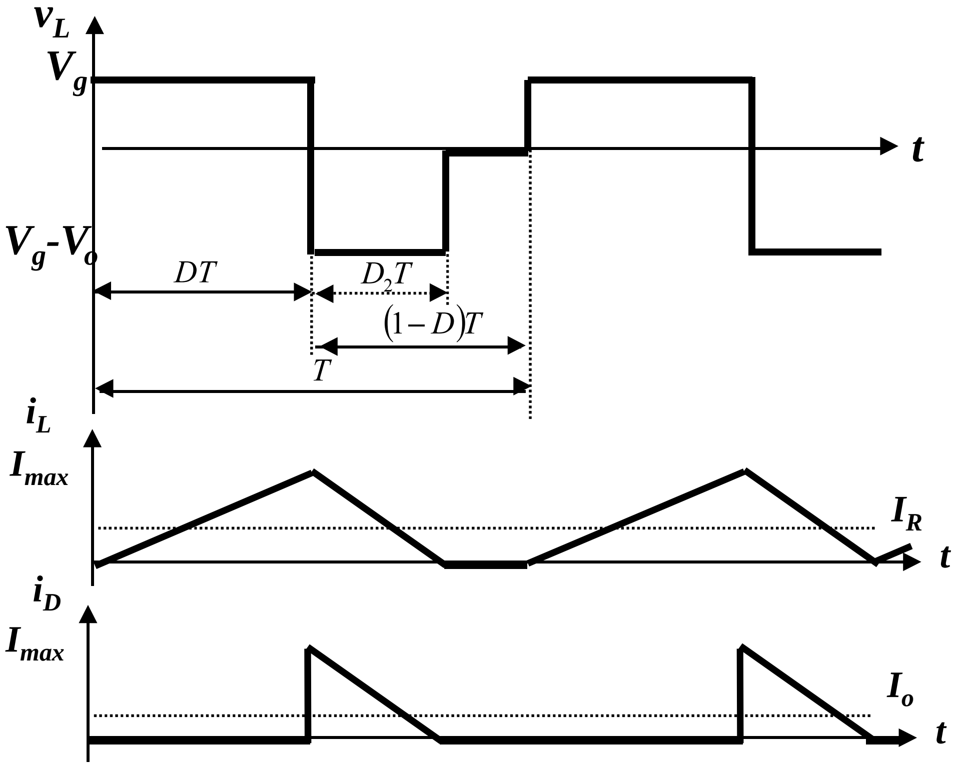
$$K = \frac{2L}{RT} \qquad K_{crit}(D) = 1 - D$$

Steady-state model in DCM

Boost DCM

$$V_o = V_g \frac{1+\sqrt{1+\frac{4D^2}{K}}}{2}$$





$$I_L > \Delta i_L \quad for \quad CCM$$

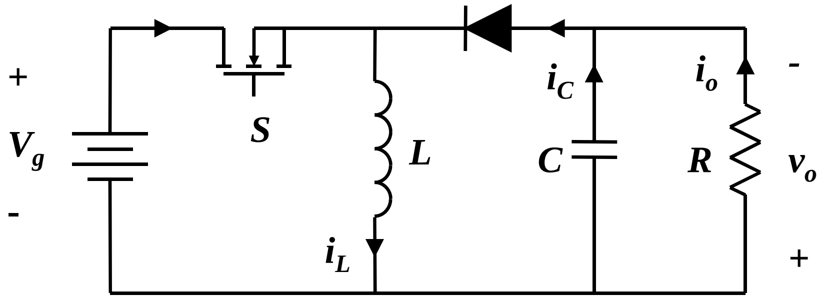
$$I_L < \Delta i_L \quad for \quad DCM$$

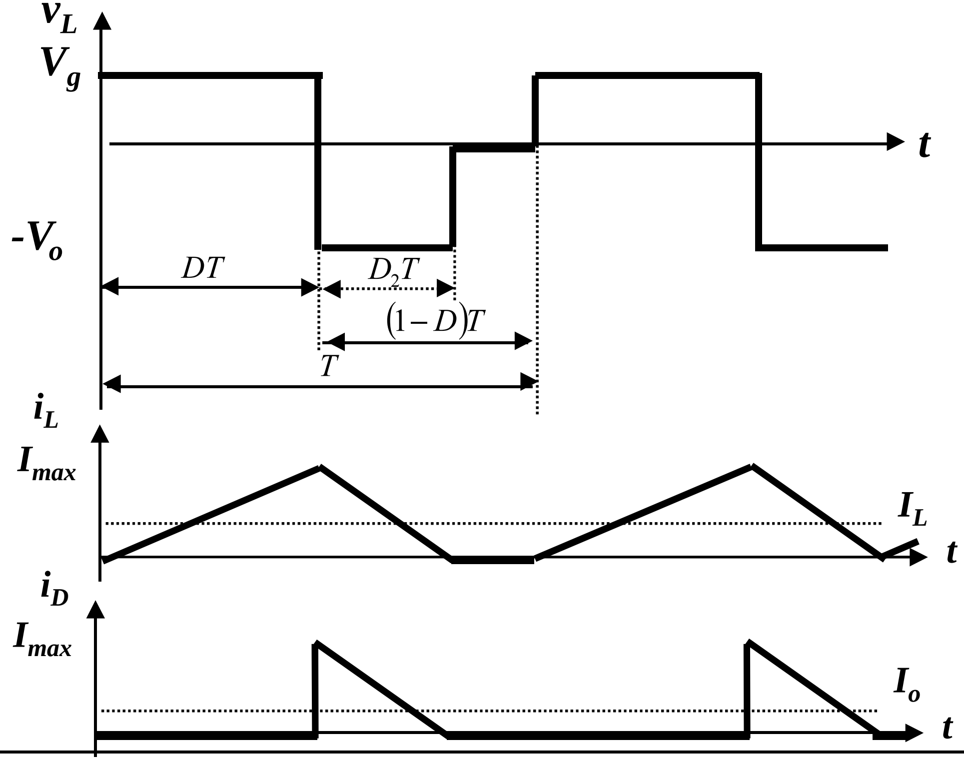
$$K = \frac{2L}{RT} \qquad K_{crit}(D) = D(1-D)^2$$

Steady-state model in DCM

Buck-Boost DCM

$$V_o = V_g \frac{D}{\sqrt{K}}$$





$$I_L > \Delta i_L \quad for \quad CCM$$
$$I_L < \Delta i_L \quad for \quad DCM$$

$$K = \frac{2L}{RT} \qquad K_{crit}(D) = (1-D)^2$$

Steady-state model in DCM

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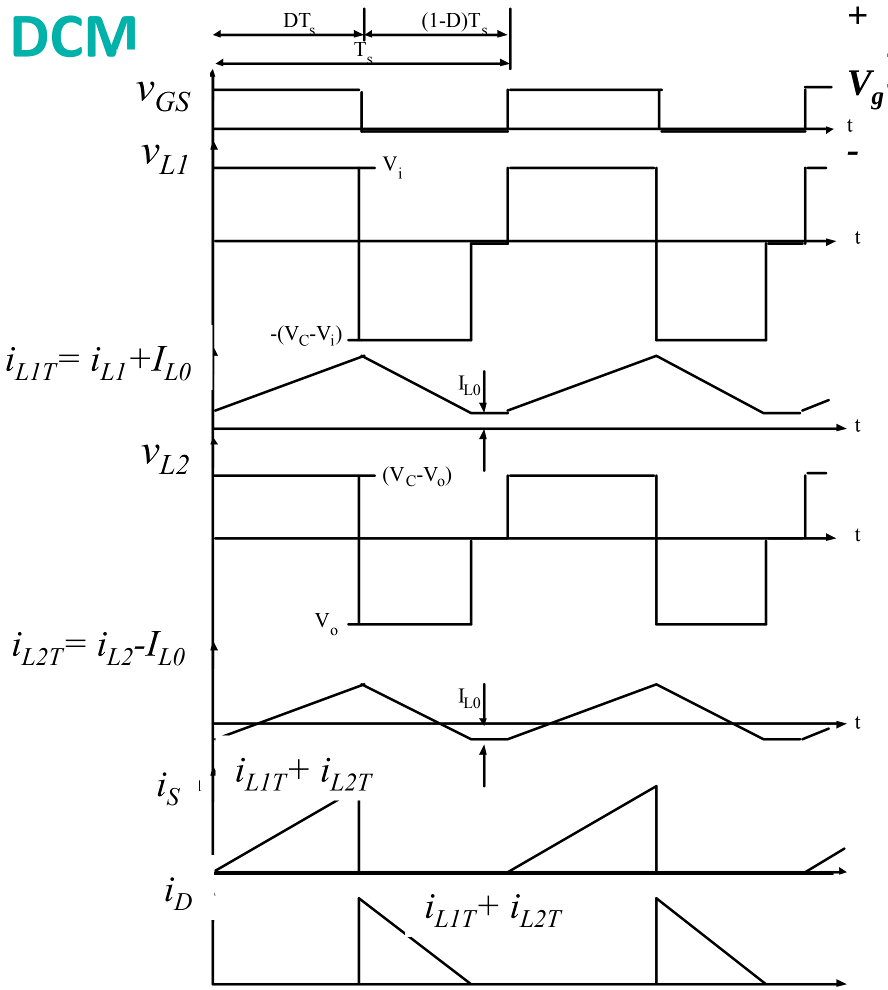
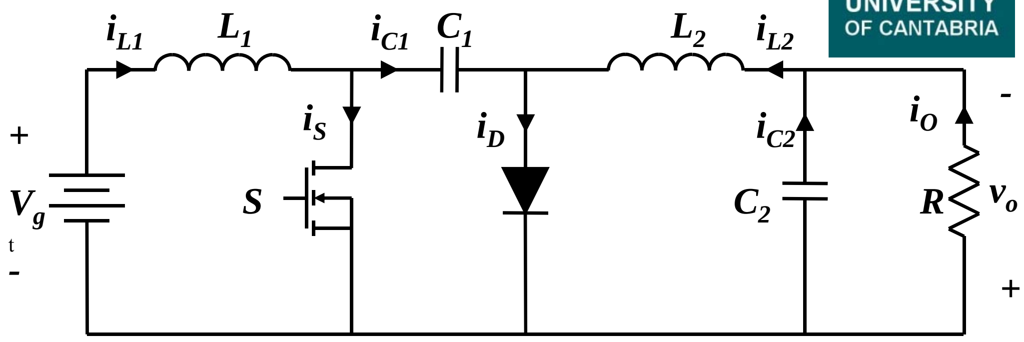
DCM





$$I_{L0} = V_g \frac{DT}{2}\left(\frac{DL_1 - D_2 L_2}{L_1 L_2}\right)$$

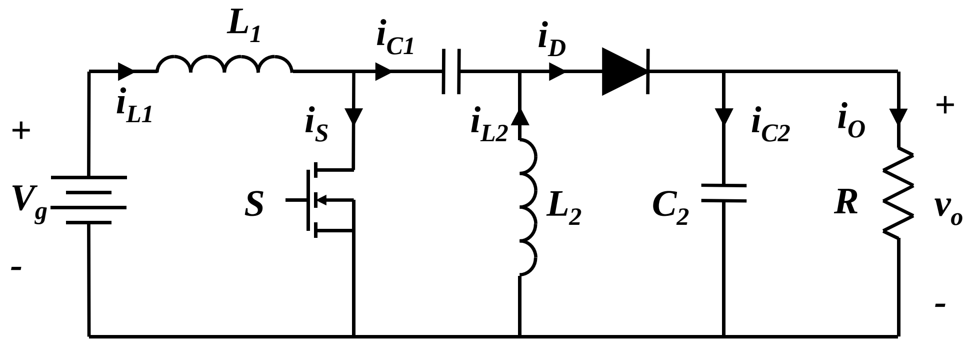
$$I_o = V_g \frac{DT}{2} D_2 \frac{1}{L_{eq}} \qquad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

$$V_g + V_o = V_{C1} \qquad K = \frac{2L_{eq}}{RT}$$

$$I_D = I_o \qquad \frac{V_o}{V_g} = \frac{D}{D_2} \qquad \frac{V_o}{V_g} = \frac{D}{\sqrt{K}}$$

Steady-state model in DCM

SEPIC DCM



$$V_g\left(D+D_2\right)=\left(V_{C1}+V\right)D_2$$

$$I_{L0}=V_g\frac{DT}{2}\left(\frac{1-\left(D+D_2\right)}{L_1}+\frac{1}{L_2}\right)$$

$$V_{C1}=V\frac{D_2}{D}$$

$$I_D=I_o=V_g\frac{DT}{2}D_2\frac{1}{L_{eq}}$$

$$L_{eq}=\frac{L_1L_2}{L_1+L_2}$$

$$V=V_g\frac{D}{D_2}$$

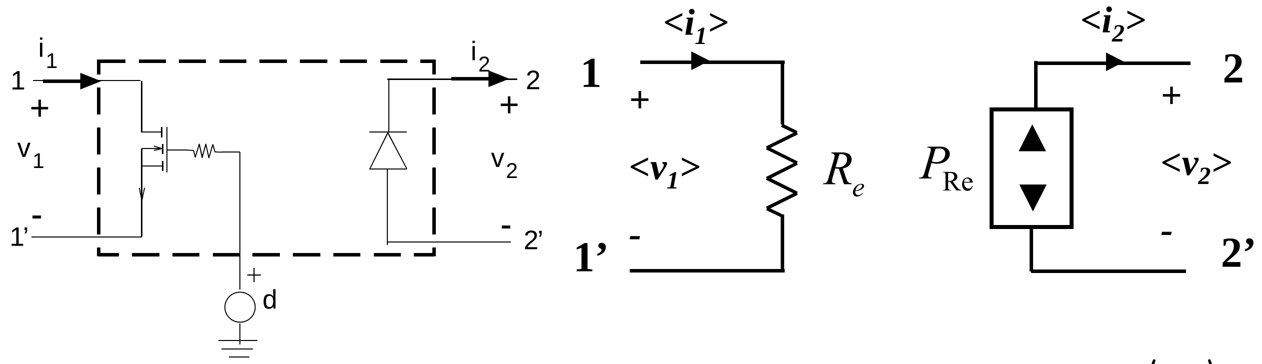
$$V_{C1}=V_g$$

$$\frac{V_o}{V_g}=\frac{D}{\sqrt{K}}$$

$$K=\frac{2L_{eq}}{RT}$$

Switch model DCM steady-state

Lossless resistance model (Boost example. The result is valid for other topologies)



$$d_2 = d\frac{\langle v_g \rangle}{\langle v \rangle - \langle v_g \rangle}$$

$$\langle v_1 \rangle = \langle v \rangle d_2 + \langle v_g \rangle (1 - d - d_2)$$

$$\langle v_2 \rangle = \langle v \rangle d + (\langle v \rangle - \langle v_g \rangle)(1 - d - d_2)$$

$$\langle i_1 \rangle = \frac{\langle v_g \rangle d^2 T}{2L}$$

$$\langle i_2 \rangle = \frac{\langle v_g \rangle d d_2 T}{2L}$$

$$\langle v_1 \rangle = \langle v_g \rangle$$

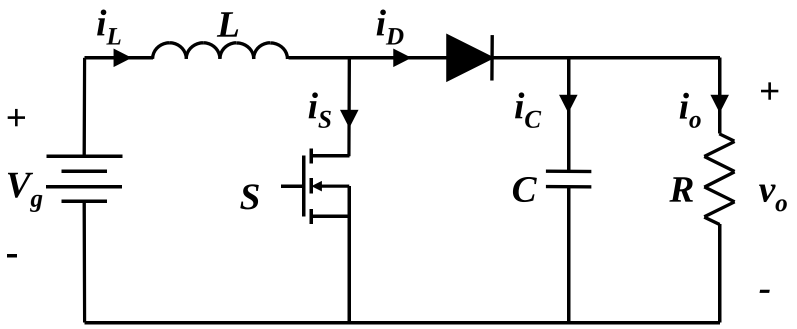
$$\langle v_2 \rangle = \langle v \rangle - \langle v_g \rangle$$

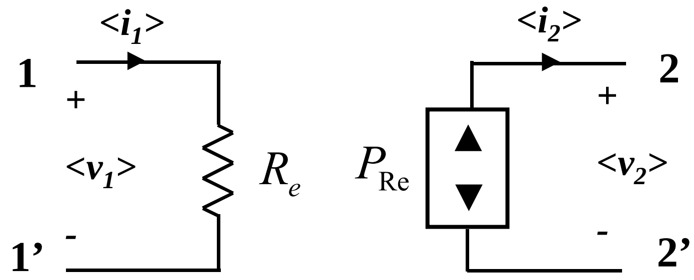
$$\langle i_2 \rangle = \frac{\langle v_g \rangle^2}{\langle v \rangle - \langle v_g \rangle}\frac{d^2 T}{2L}$$

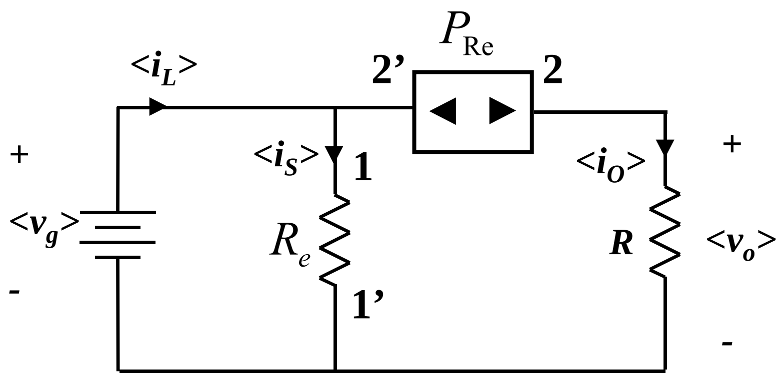
$$R_e = \frac{2L}{d^2 T}$$

Switch model DCM steady-state

Boost example

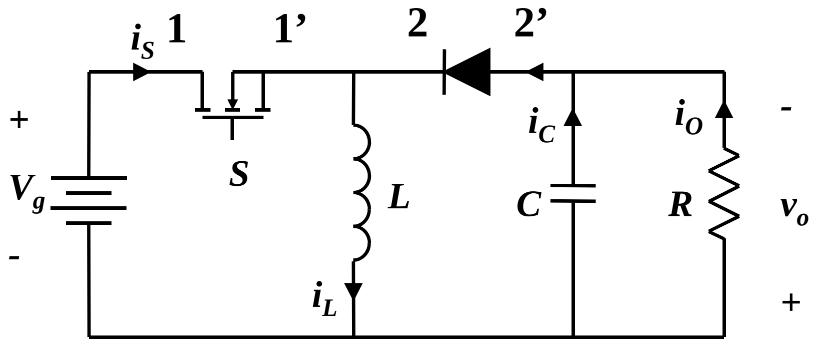


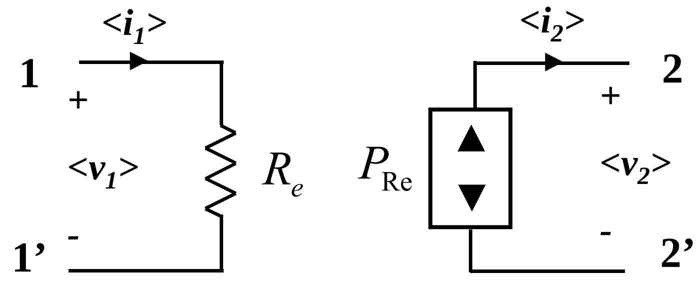


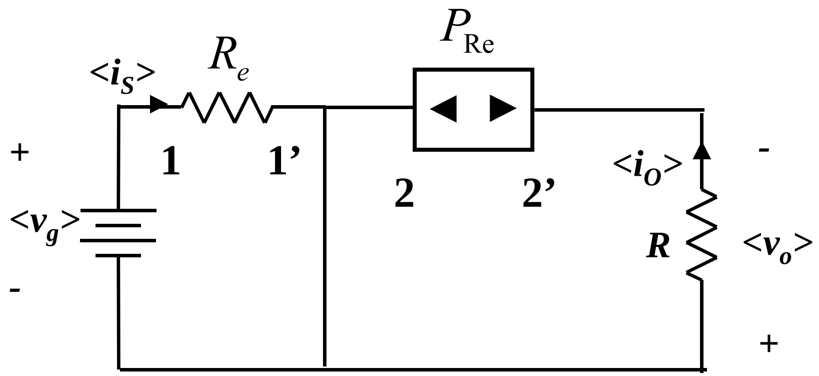


Switch model DCM steady-state

Buck-Boost example

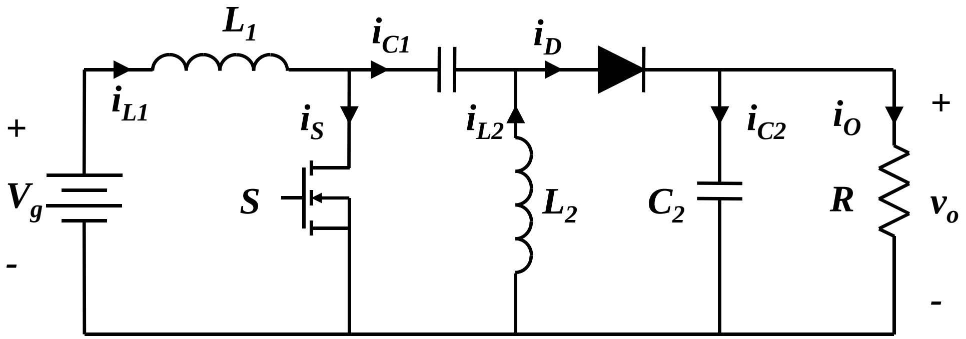


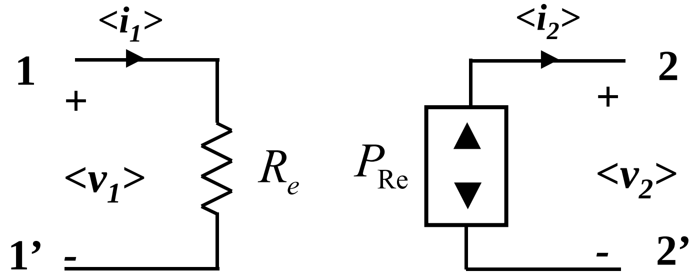


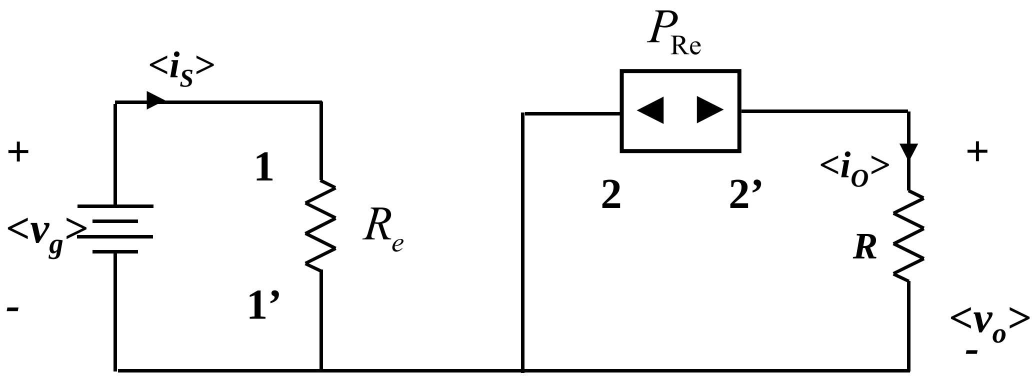


Switch model DCM steady-state

SEPIC example







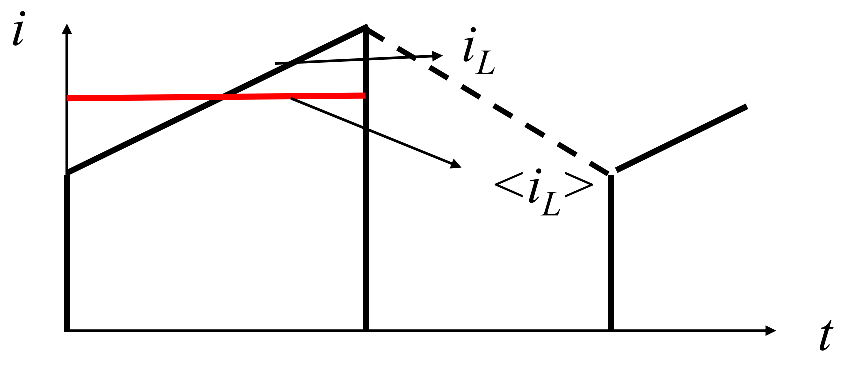
$$R_e = \frac{2L_{eq}}{D^2T} \qquad \frac{V_g^2}{R_e} = P_{R_e} = \frac{V_o^2}{R}$$

$$\frac{V_o}{V_g} = \sqrt{\frac{R}{R_e}} \qquad \frac{V_o}{V_g} = \sqrt{\frac{D^2TR}{2L_{eq}}}$$

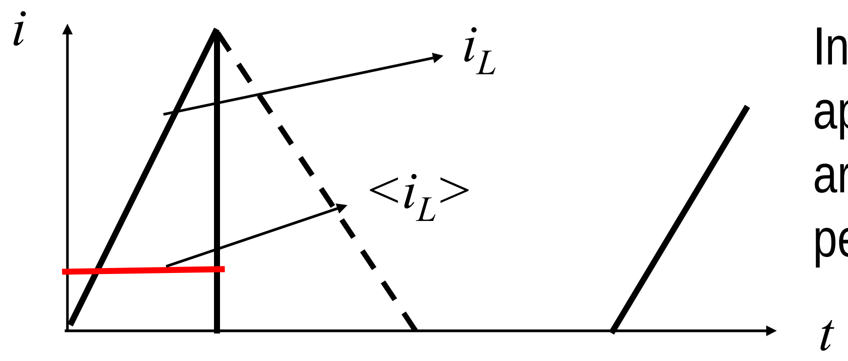
$$\frac{V_o}{V_g} = \frac{D}{\sqrt{k}} \qquad k = \frac{2L_{eq}}{RT} \qquad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

In CCM the small-ripple approximation is used to calculate average values

Buck Boost example



The amps-second area is equivalent if it is calculated with i_L or with the low-ripple approximation $< i_L >$



In the DCM case the low-ripple is a poor approximation to calculate the amps-second area. So the integral i_L over the switching period should be computed.

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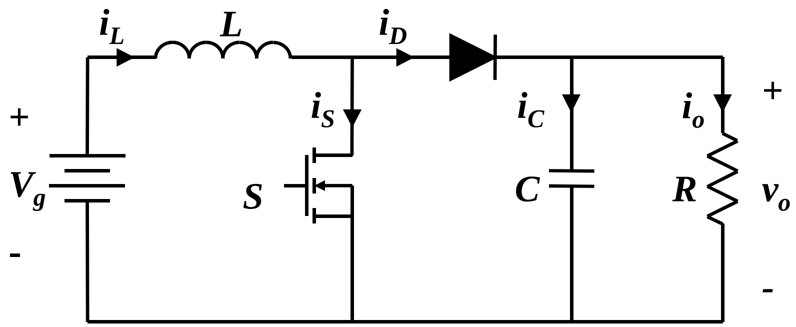
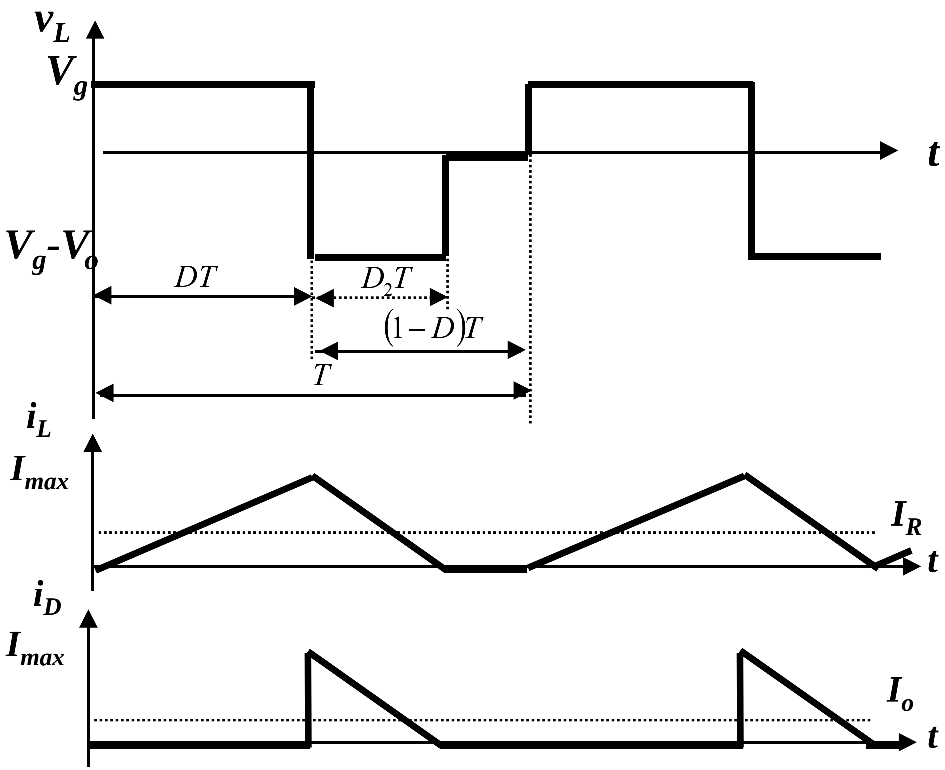
Boost example





$$\dot{x} = A_1 x + B_1 v_g \qquad [0,\ dT]$$

$$\dot{x} = A_2 x + B_2 v_g \qquad [dT,\ (d{+}d_2)T]$$

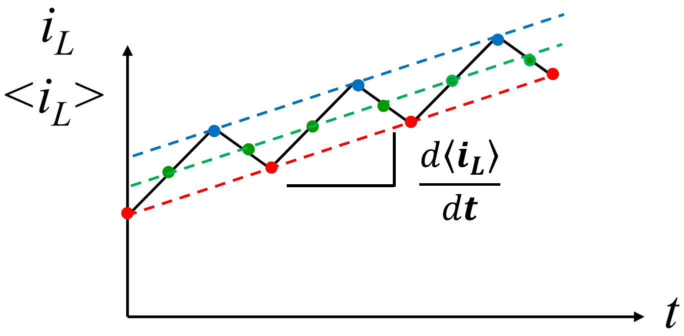
$$\dot{x} = A_3 x + B_3 v_g \qquad [(d{+}d_2)T,\ T]$$

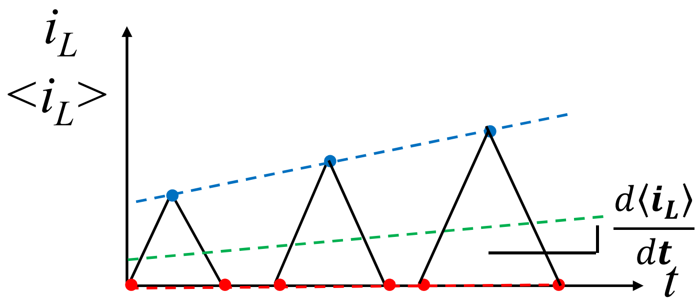
Dynamic modeling DC – DC converters in DCM

The model of the converter is obtained by averaging the state variables over the switching period.

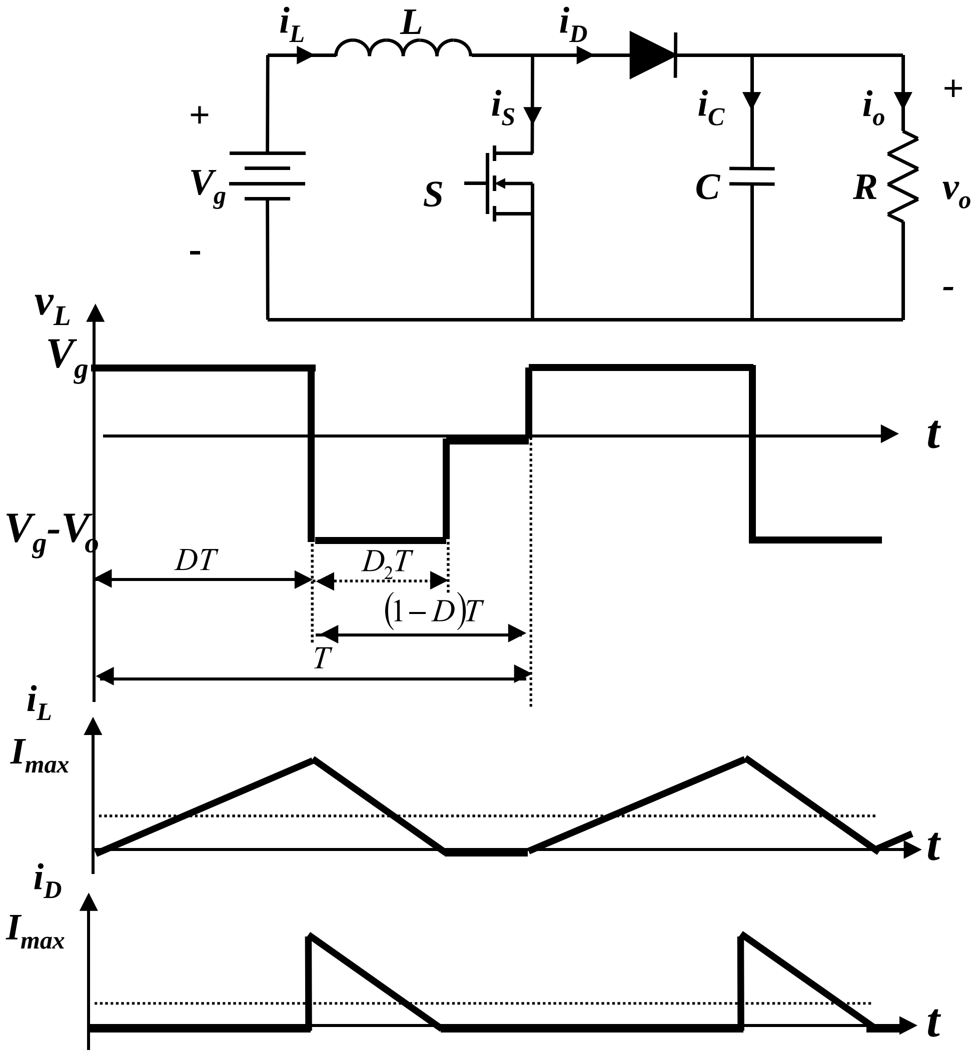
a) Methods to obtain the averaged functions in CCM may not be applied in DCM.

b) Observe a transient with time response larger (slower) than the switching period. In CCM the peak, valley, mid or any other value has the same variation over time. This is not the case in DCM, except in steady-state.





Boost example



Average

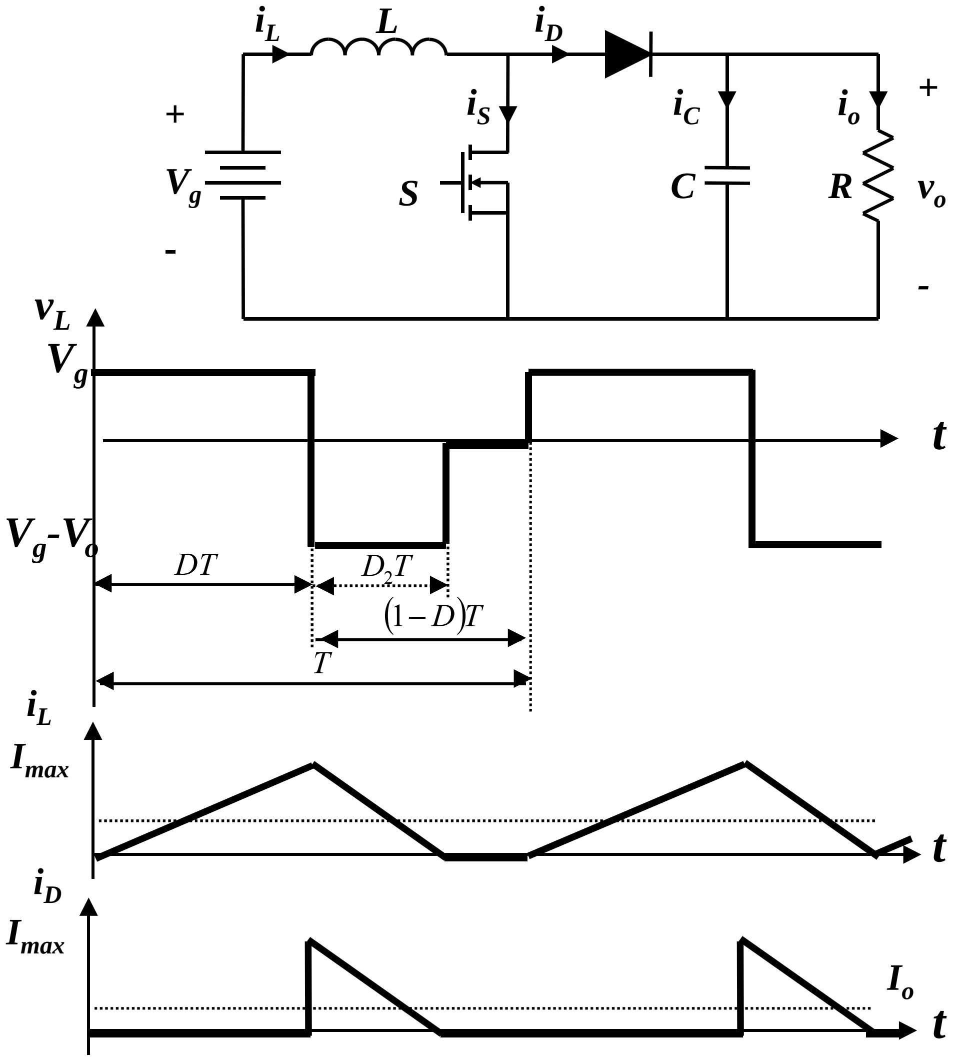
$$\frac{d\langle i_L\rangle}{dt} = \frac{(d+d_2)\langle v_g\rangle - d_2\langle v\rangle}{L}$$

$$\frac{d\langle v_c\rangle}{dt} = \frac{\frac{1}{T}\int_{dT}^{(d+d_2)T} i_L dt - \frac{\langle v\rangle}{R}}{C}$$

$$¿¿d_2??$$

Dynamic modeling DC – DC converters in DCM

Boost example steady-state



[0, dT]

$$\frac{di_L}{dt} = \frac{v_g}{L} \qquad \frac{dv}{dt} = \frac{-\frac{v}{R}}{C}$$

[dT, (d+d_2)T]

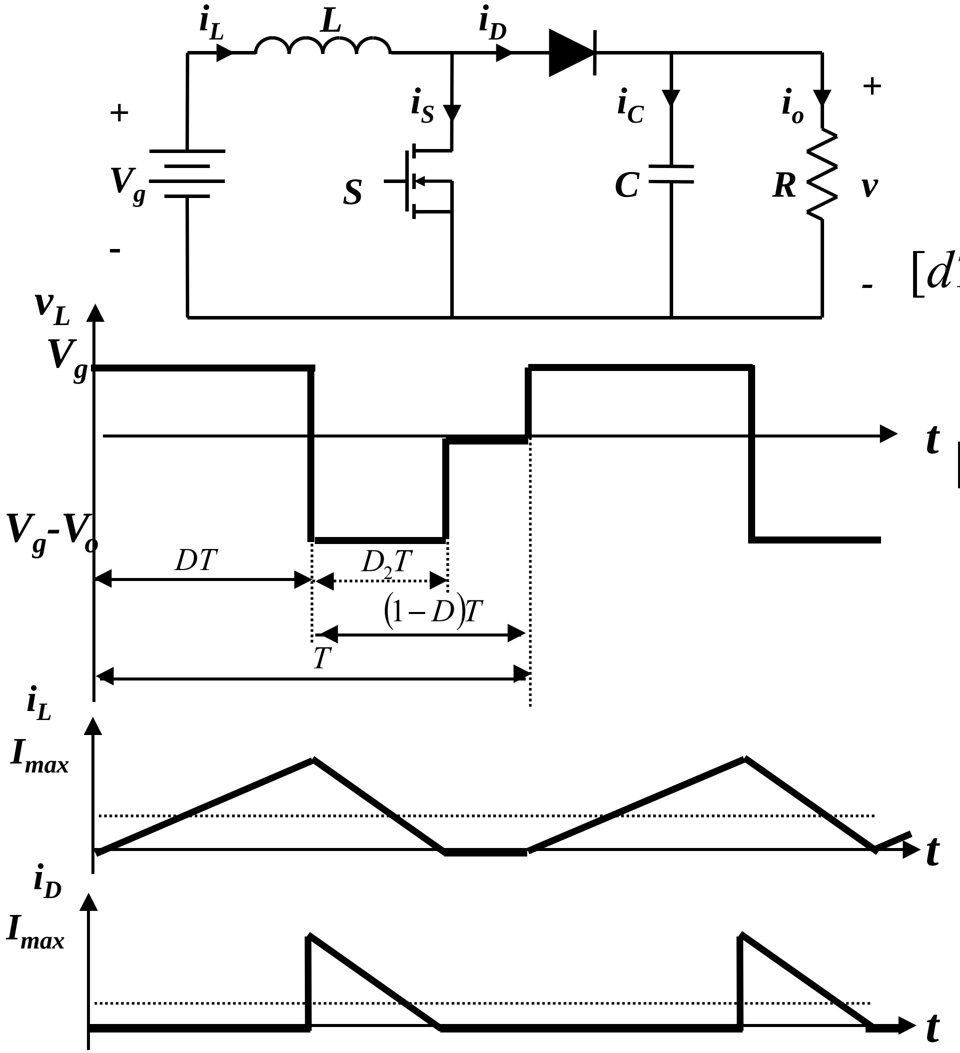
$$\frac{di_L}{dt} = \frac{v_g - v}{L} \qquad \frac{dv}{dt} = \frac{i_L - \frac{v}{R}}{C}$$

[(d+d_2)T, T]

$$\frac{di_L}{dt} = 0 \qquad \frac{dv}{dt} = \frac{-\frac{v}{R}}{C}$$

Average paradox

In DCM, the current is zero at the beginning and end of each switching period



$[0, dT]$
$$\frac{di_L}{dt} = \frac{v_g}{L}$$

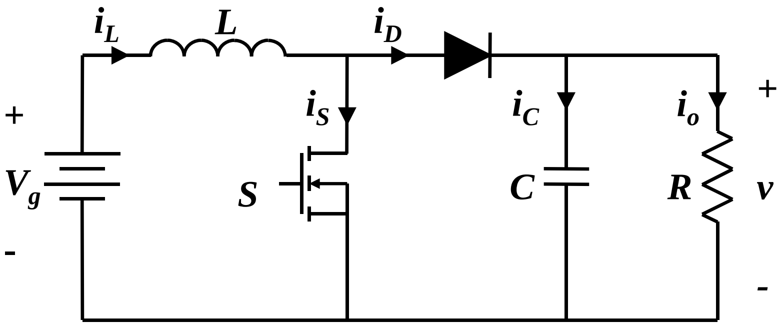
$[dT, (d+d_2)T]$
$$\frac{di_L}{dt} = \frac{v_g - v}{L}$$

$[(d+d_2)T, T]$
$$\frac{di_L}{dt} = 0$$

$$\frac{\langle v_g \rangle}{L} d + \frac{\langle v_g \rangle - \langle v \rangle}{L} d_2 = 0$$

$$d_2 = \frac{d \langle v_g \rangle}{\langle v \rangle - \langle v_g \rangle}$$

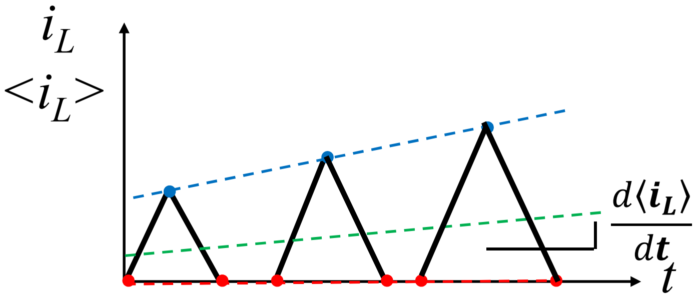
This method would reach the same conclusion in transient state



$[0, dT]$ $$\frac{di_L}{dt} = \frac{v_g}{L}$$

$[dT, (d+d_2)T]$ $$\frac{di_L}{dt} = \frac{v_g - v}{L}$$

$[(d+d_2)T, T]$ $$\frac{di_L}{dt} = 0$$



$$\frac{\langle v_g \rangle}{L} d + \frac{\langle v_g \rangle - \langle v \rangle}{L} d_2 = 0$$

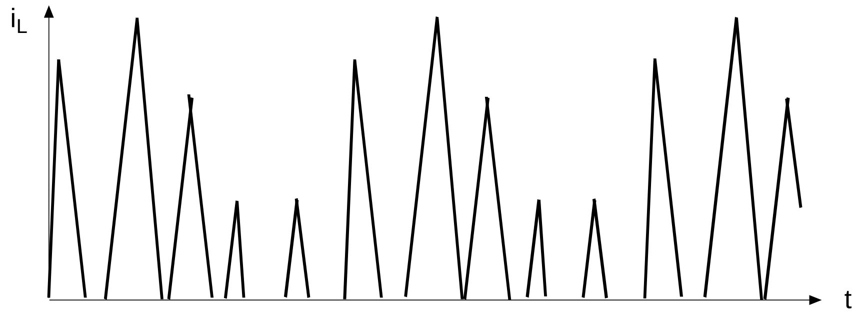
$$d_2 = \frac{\langle v_g \rangle}{\langle v \rangle - \langle v_g \rangle}$$

$$¿¿ \frac{d\langle i_L \rangle}{dt} ??$$

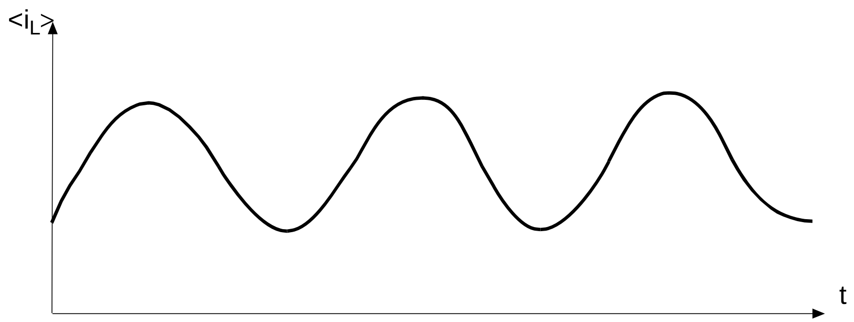
Average paradox

Use $$d_2 = \frac{d\langle v_g \rangle}{\langle v \rangle - \langle v_g \rangle}$$ ➔ $$¿¿\frac{d\langle i_L \rangle}{dt}??$$

means to assume <i_L> in steady-state



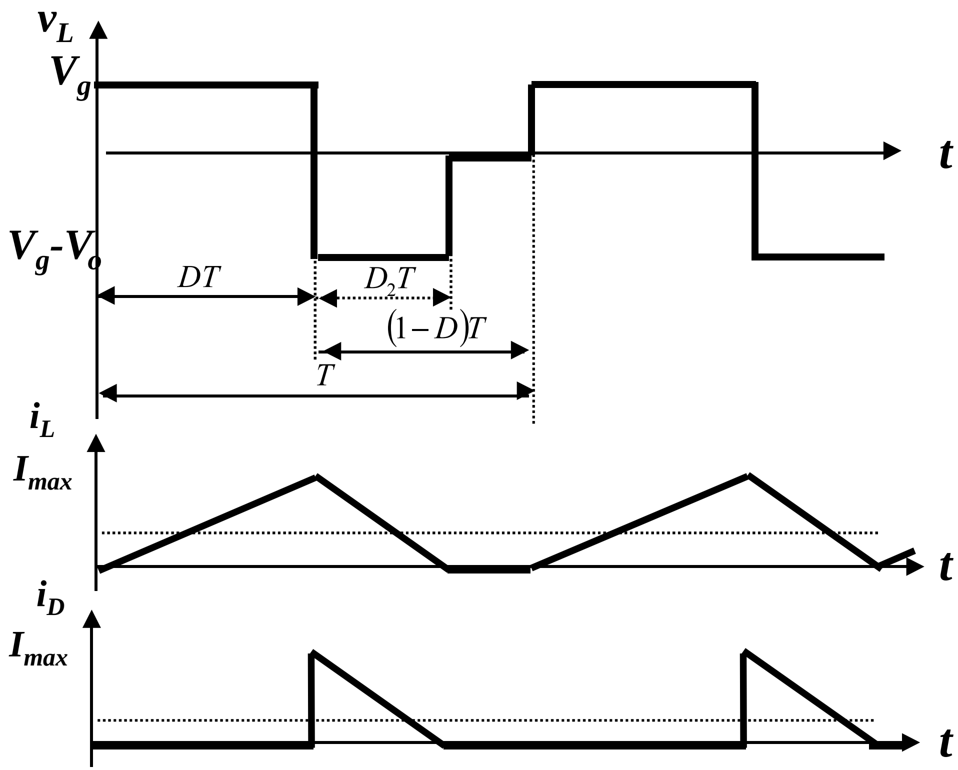
and it actually is



Average paradox

If the inductor current has a low frequency component there will be also a low frequency voltage component across the inductor.

However, a sampler of average voltage at the end of each switching period kT acquires $<v_L>(kT)=0$



Average paradox

a) We have assumed <i_L> in steady-state

b) We have not calculated the continuous function <i_L>, but we have sampled i_L at each kT.

c) We have assumed that the sampled value <i_L> is constant in the whole switching period (zero-order hold).

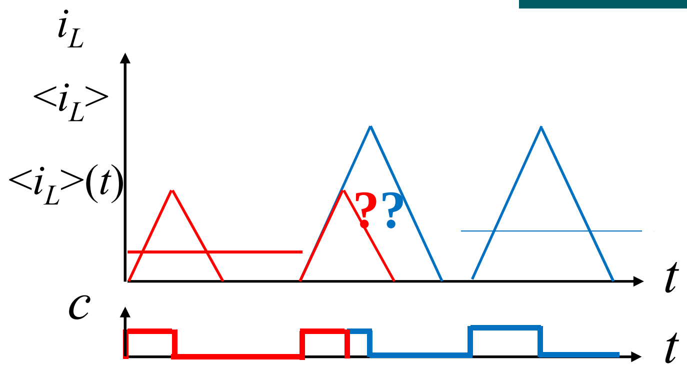
d) We have sampled i_L with a sampled frequency equal the main frequency component of the function.

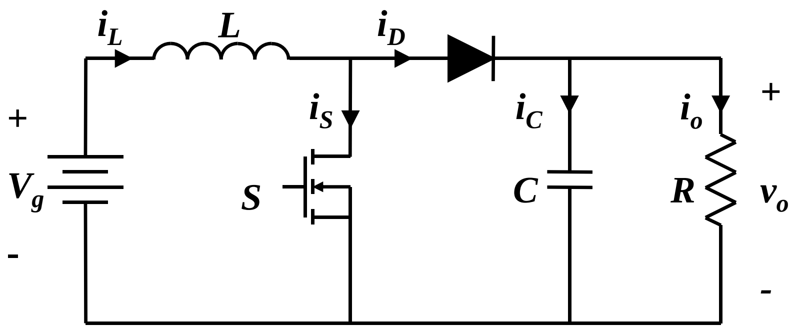
e) We have assumed that the voltage does not change within the switching period (low voltage ripple approximation).

Dynamic reduced order model

The Boost converter is used as example

If it is assumed that the inductor causes no delay to $<i_L>$, the assumption means a quasi-static approximation, so that the steady-state restriction is used and the order of the model is reduced by 1





$$\frac{d\langle v_o\rangle}{dt} = \frac{d_2\langle i_L\rangle}{C} - \frac{\langle v_o\rangle}{RC}$$

$$\langle i_L\rangle = \frac{1}{T}\int_{t_o}^{t_o+T} i_L\,dt$$ the value is assigned to the period

$$\langle i_L\rangle(t) = \frac{1}{T}\int_{t_o-\frac{T}{2}}^{t_o+\frac{T}{2}} i_L(\tau)\,d\tau$$ continuous average function

Dynamic reduced order model

Boost converter example. Model calculation without using the i_L low-ripple approximation

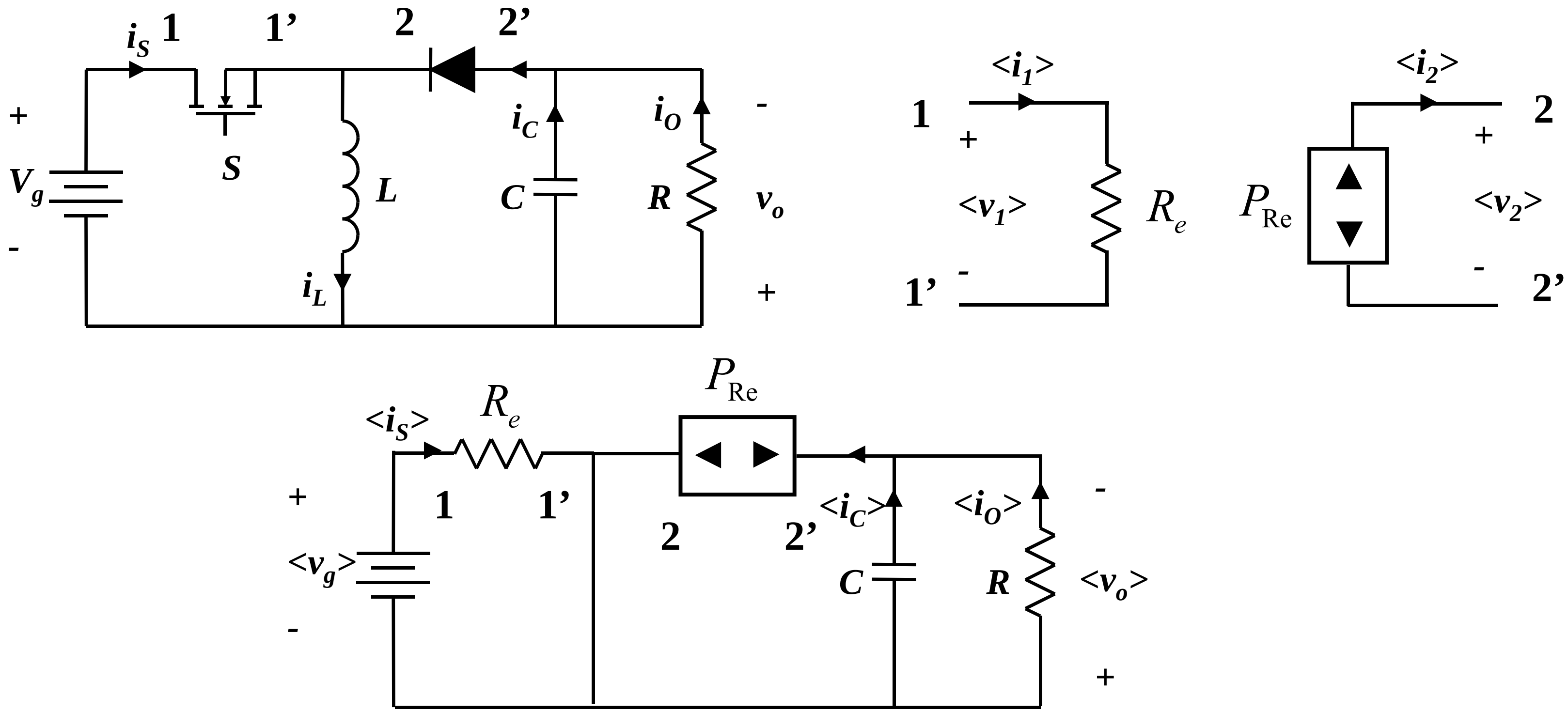
$$d_2 = d\,\frac{\langle v_g\rangle}{\langle v\rangle - \langle v_g\rangle}$$

$$\langle i_c\rangle = C\frac{d\langle v\rangle}{dt} = \frac{1}{T}\left\{-\frac{\langle v\rangle}{R} + \int_{dT}^{(d+d_2)T}\left[\frac{\langle v_g\rangle}{L}dT + \frac{\langle v_g\rangle - \langle v\rangle}{L}(t-dT)\right]dt\right\}$$

$$\frac{d\langle v\rangle}{dt} = \frac{\langle v_g\rangle^2 d^2 T}{2LC\left(\langle v\rangle - \langle v_g\rangle\right)} - \frac{\langle v\rangle}{RC}$$

Reduced order averaged dynamic model

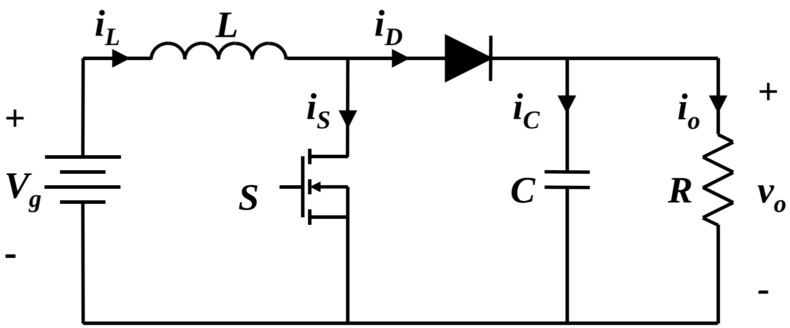
Buck-Boost example

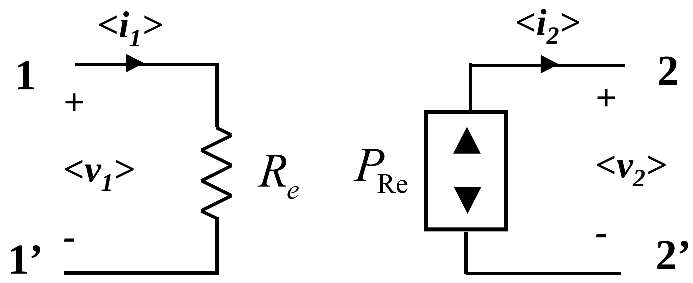


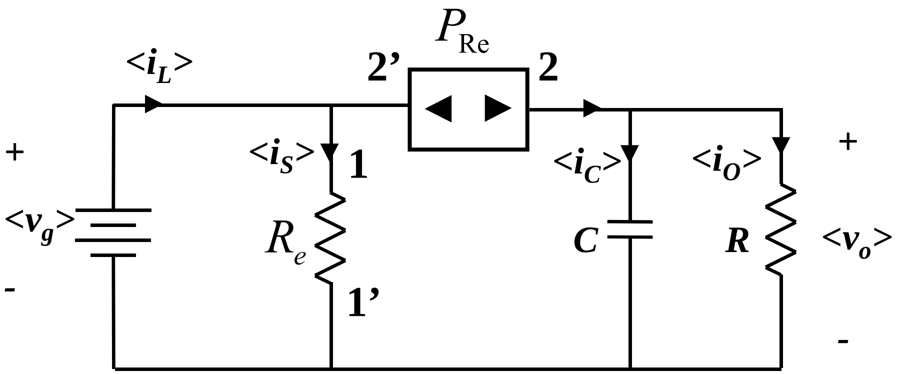
L removed from the model

Reduced order averaged dynamic model

Boost example







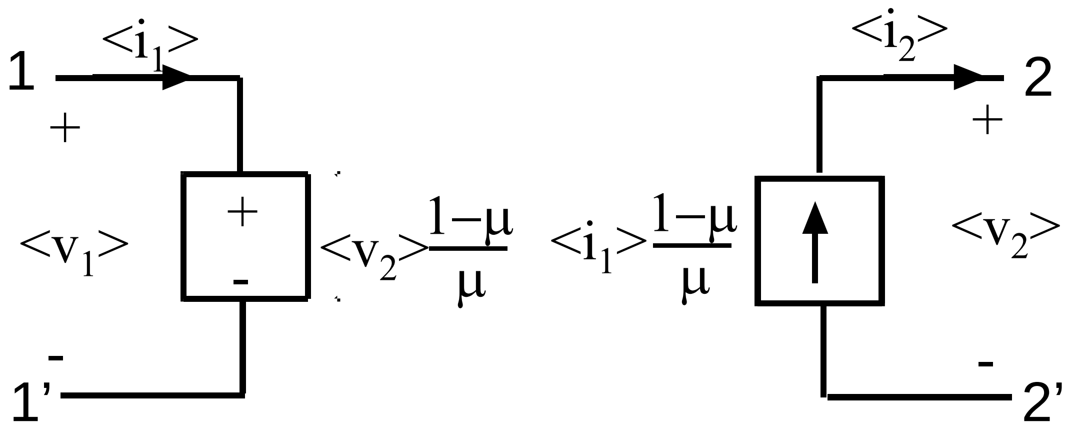
L removed from the model

Switch model compatible CCM-DCM

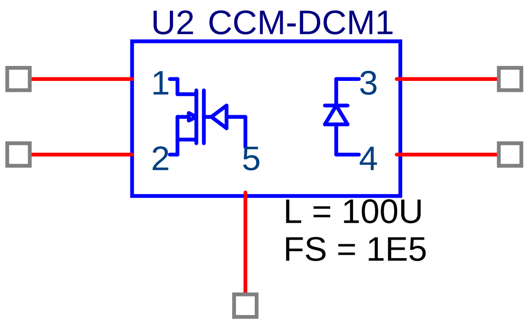
Lossless resistance model

$$\langle v_1 \rangle = R_e \langle i_1 \rangle \qquad R_e = \frac{2L}{d^2 T} \qquad \langle v_2 \rangle = \langle v_1 \rangle \frac{\mu}{1-\mu} \qquad \langle v_2 \rangle = R_e \langle i_1 \rangle \frac{\mu}{1-\mu}$$

$$\mu = \frac{\langle v_2 \rangle}{R_e \langle i_1 \rangle + \langle v_2 \rangle} \qquad \mu = \frac{d^2}{\frac{2Lf_s \langle i_1 \rangle}{\langle v_2 \rangle} + d^2}$$



Switch model compatible CCM-DCM



Ref. 8:
R.W. Erickson, D. Maksimovic. *Fundamentals of Power Electronics 2nd edition*. Kluwer Academic Publishers. 2001

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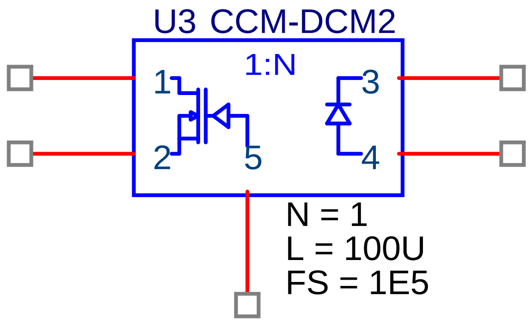
************************************************************
* MODEL: CCM-DCM1
* Application: two-switch PWM converters, CCM or DCM
* Limitations: ideal switches, no transformer
************************************************************
* Parameters:
*      L=equivalent inductance for DCM
*      fs=switching frequency
************************************************************
* Nodes:
* 1: transistor positive (drain of an n-channel MOS)
* 2: transistor negative (source of an n-channel MOS)
* 3: diode cathode
* 4: diode anode
* 5: duty cycle control input
************************************************************
.subckt CCM-DCM2 1  2  3  4  5
+params: L=100u fs=1E5
Et 1 2 value={(1-v(u))*v(3,4)/v(u)}
Gd 4 3 value={(1-v(u))*i(Et)/v(u)}
Ga 0 a value={MAX(i(Et),0)}
** In case of convergence problems, the alternative Ga
** expression may work better:
* Ga 0 a value={i(Et)}
Va a b
Ra b 0 10K
Eu u 0 table {MAX(v(5),
+ v(5)*v(5)/(v(5)*v(5)+2*L*fs*i(Va)/v(3,4)))}(0 0) (1 1)
.ends
************************************************************
```

Switch model compatible CCM-DCM



Ref. 8:
R.W. Erickson, D. Maksimovic.
Fundamentals of Power Electronics 2nd edition. Kluwer Academic Publishers. 2001

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*******************************************************************
* MODEL: CCM-DCM2
* Application: two-switch PWM converters, CCM or DCM
*              with (possibly) transformer
* Limitations: ideal switches
**********************************************************
* Parameters:
*      L=equivalent inductance for DCM,
*        referred to primary
*      fs=switching frequency
*      n=transformer turns ratio 1:n (primary:secondary)
**********************************************************
* Nodes:
* 1: transistor positive (drain of an n-channel MOS)
* 2: transistor negative (source of an n-channel MOS)
* 3: diode cathode
* 4: diode anode
* 5: duty cycle control input
**********************************************************
.subckt CCM-DCM2 1  2  3  4  5
+params: L=100u fs=1E5 n=1
Et 1 2 value={(1-v(u))*v(3,4)/v(u)/n}
Gd 4 3 value={(1-v(u))*i(Et)/v(u)/n}
* Ga 0 a value={MAX(i(Et),0)}
Ga 0 a value={i(Et)}
Va a b
Ra b 0 10K
Eu u 0 table {MAX(v(5),
+ v(5)*v(5)/(v(5)*v(5)+2*L*n*fs*i(Va)/v(3,4)))}(0 0) (1 1)
.ends
```
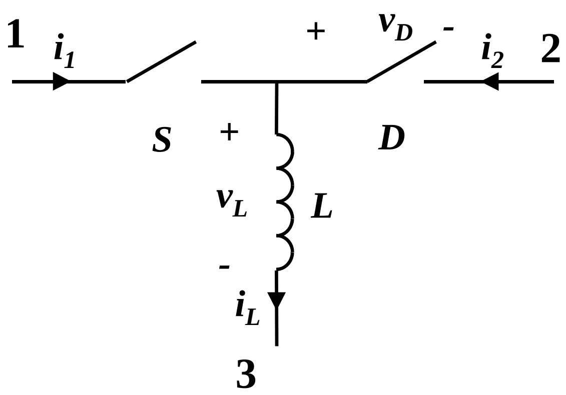

Complete order dynamic model

Generalized equivalent circuit



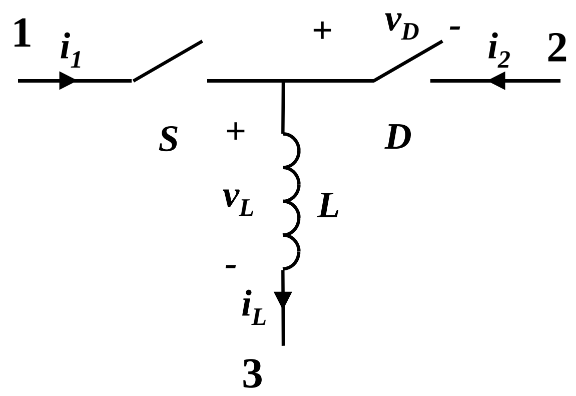
$$d_2 = -\frac{\langle v_{13}\rangle}{\langle v_{23}\rangle} d$$

$$\langle i_1\rangle = \frac{d^2 \langle v_{13}\rangle T}{2L}$$

$$\langle i_2\rangle = -\frac{d^2 \langle v_{13}\rangle^2 T}{2L\langle v_{23}\rangle}$$

Complete order dynamic model

Same switch model, but L is not removed from the model



$$L\frac{di_L}{dt} = \langle v_{12} \rangle + \langle v_{23} \rangle$$

$0 < t < dT$: S ON, D OFF

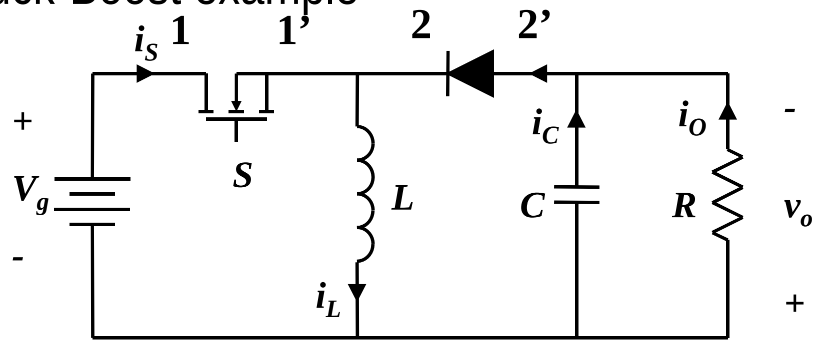
$$L\frac{di_L}{dt} = \langle v_{23} \rangle$$

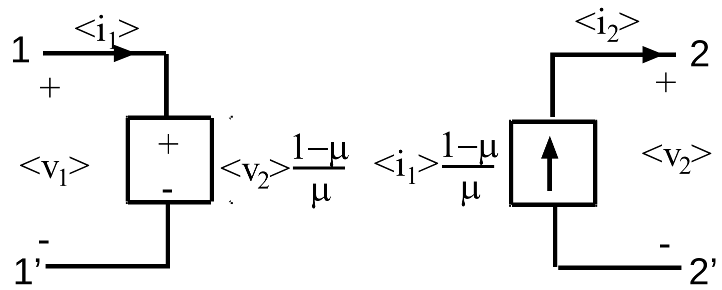
$dT < t < (d+d_2)T$: S OFF, D ON

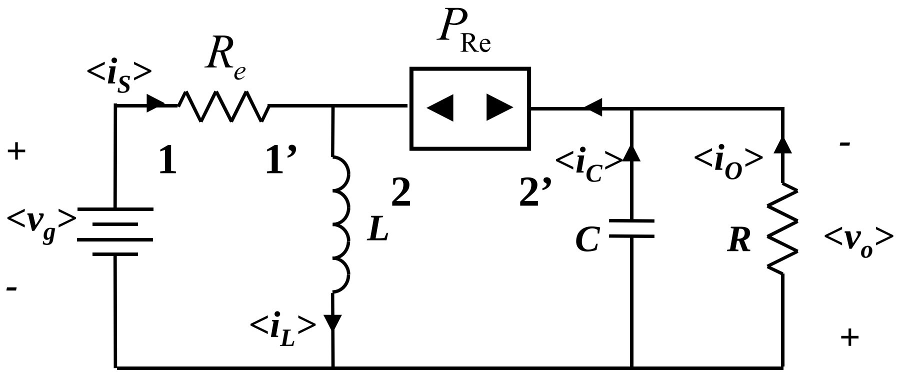
$$i_L = 0$$

$(d+d_2)T < t < T$: S OFF, D OFF

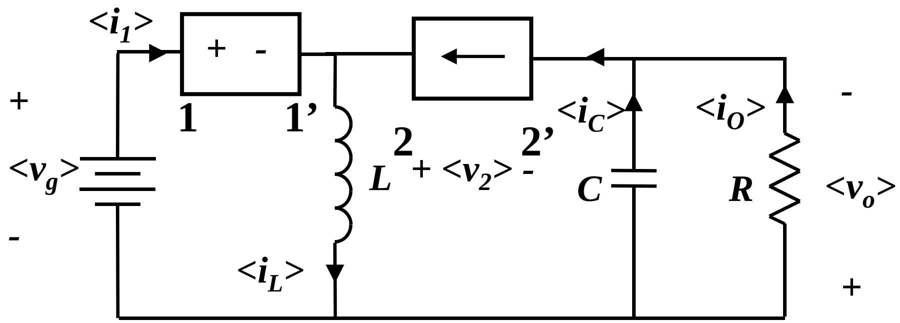
Buck-Boost example







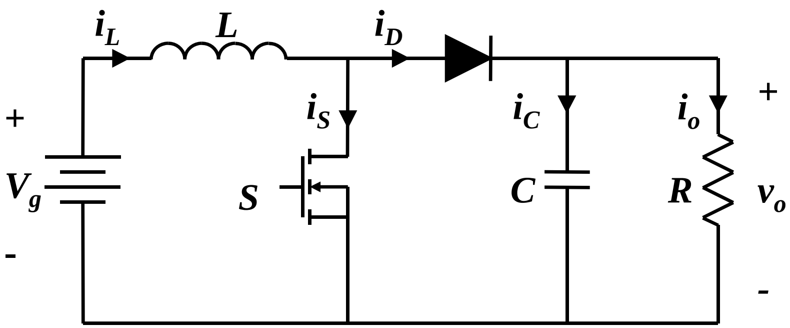
$$\langle v_1\rangle = \langle v_2\rangle \frac{1-\mu}{\mu} \qquad \langle i_2\rangle = \langle i_1\rangle \frac{1-\mu}{\mu}$$

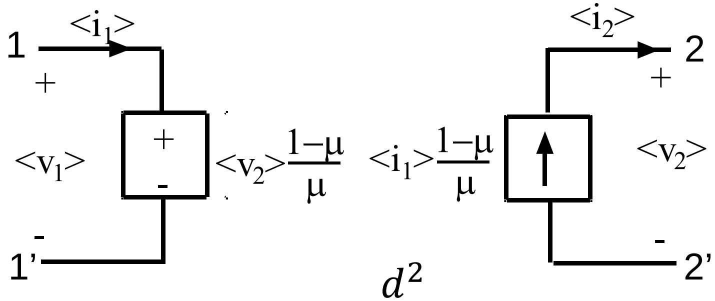


$$\mu = \frac{d^2}{\dfrac{2Lf_s\langle i_1\rangle}{\langle v_2\rangle} + d^2}$$

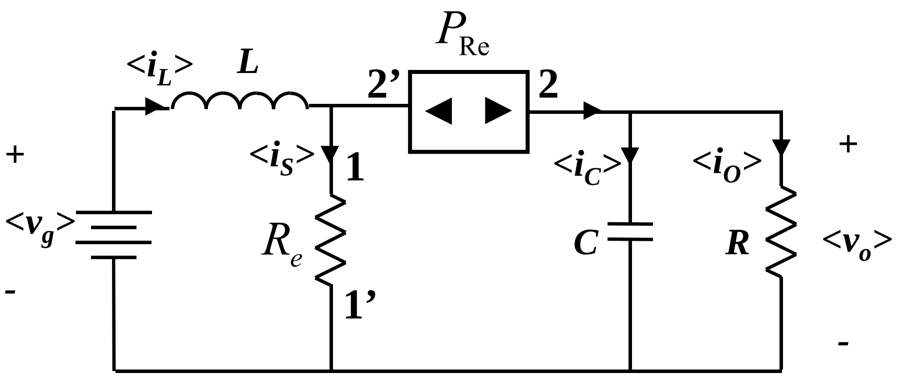
Complete order dynamic model

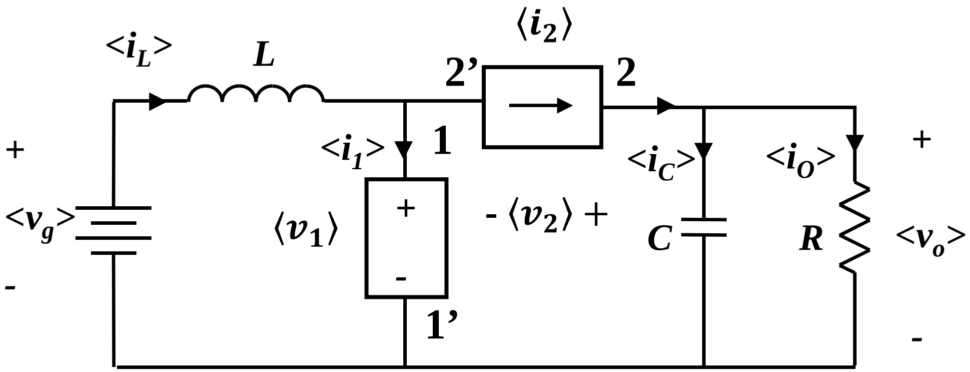
Boost example





$$\mu = \frac{d^2}{\frac{2Lf_s\langle i_1\rangle}{\langle v_2\rangle} + d^2}$$

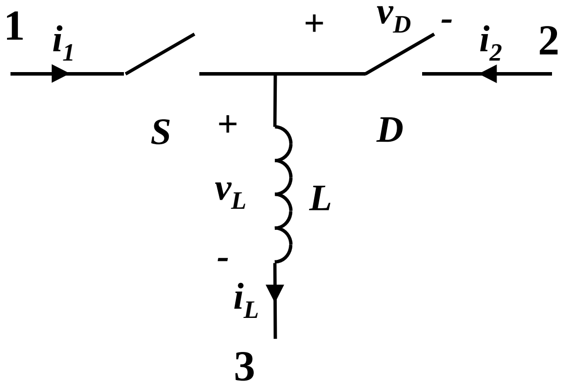




$$\langle v_1\rangle = \langle v_2\rangle \frac{1-\mu}{\mu} \qquad \langle i_2\rangle = \langle i_1\rangle \frac{1-\mu}{\mu}$$

Complete order dynamic model

This model partially uses $\langle v_L \rangle = 0$. The average output voltage does not change in each switching period



$$d_2 = -\frac{\langle v_{13} \rangle}{\langle v_{23} \rangle} d = \frac{\langle v_{12} \rangle - \langle v_D \rangle}{\langle v_D \rangle} d$$

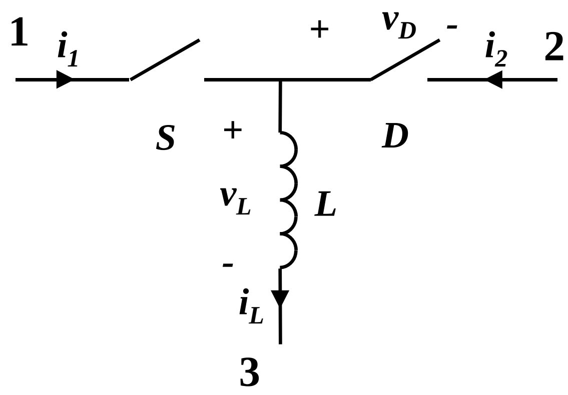
same expression as in the reduced order!

L is included in the model. It allows $\langle i_L \rangle$ to change in different switching periods, but not within the switching period. The average output voltage does not change within the switching period.

Complete order dynamic model

d_2 is calculated with $<v_L>=0$ but it means an improvement over the reduced order.

Buck-Boost



$$\langle i_L \rangle = \langle i_1 \rangle + \langle i_2 \rangle$$

L is included in the model. It allows $<i_L>$ to change in different switching periods, but not within the switching period. The average output voltage does not change within the switching period.

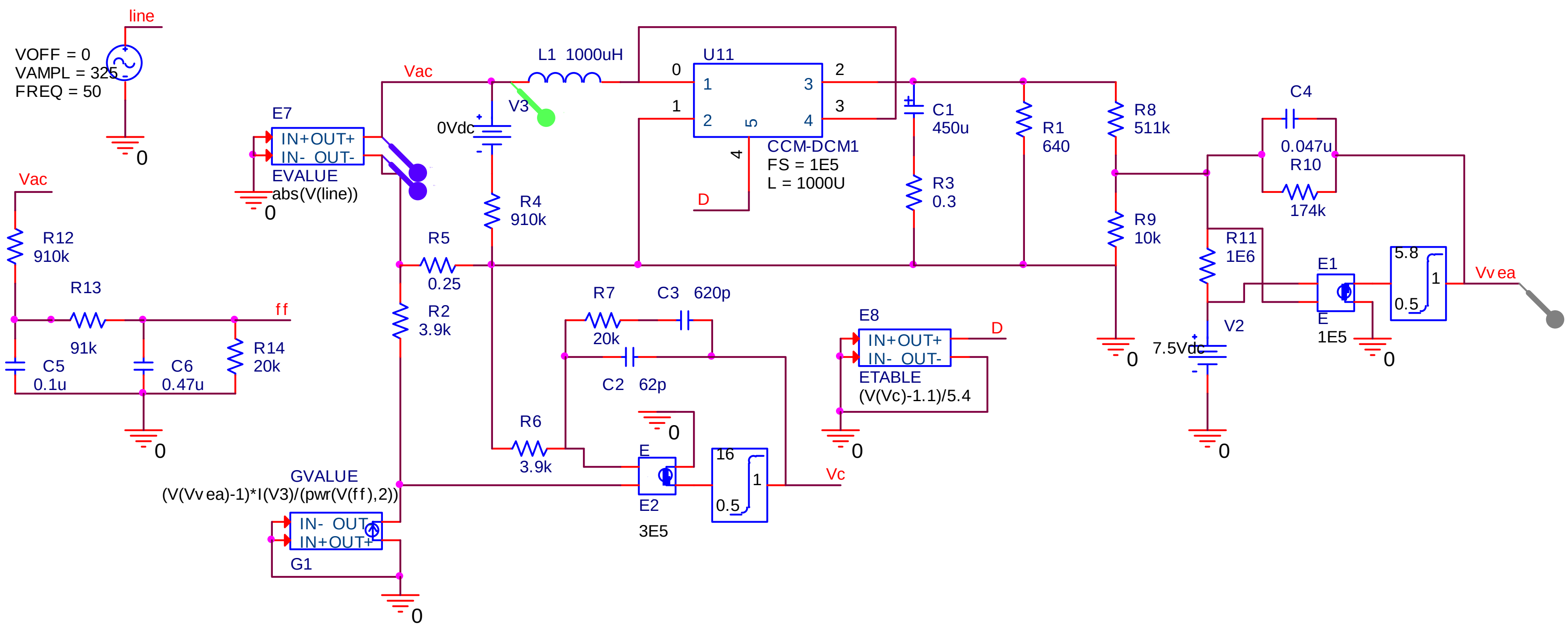
$$d_2 = d\frac{\langle v_g \rangle}{\langle v_o \rangle}$$ **?**

- More accurate than the reduced order model
- Not as accurate as the continuous model

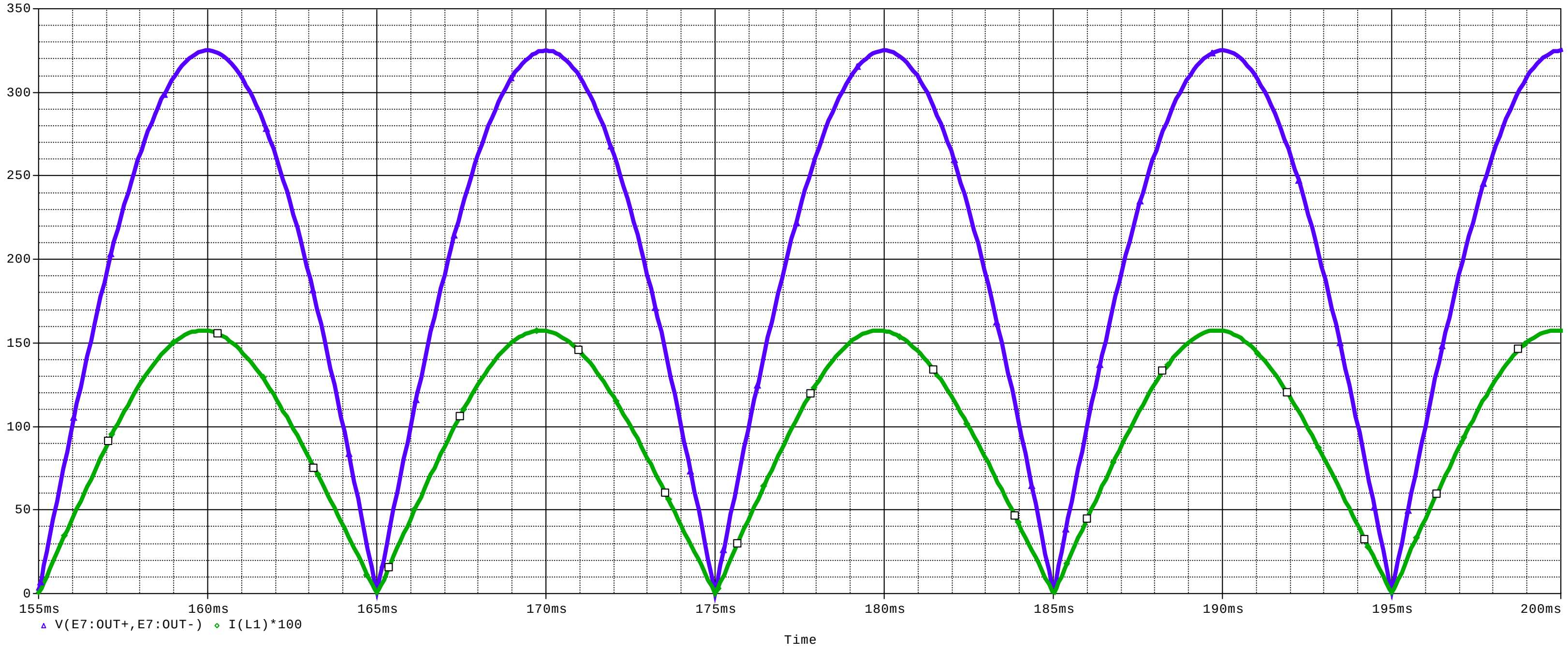
Complete order dynamic model

CCM-DCM Dynamic average model with ASM

Boost PFC with UC 3854 controller model example

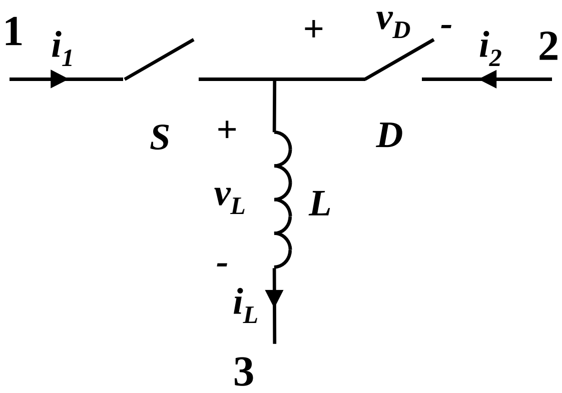


PFC with UC 3854 controller model example



Continuous dynamic model

The objective is to obtain d_2 without using $\langle v_o \rangle$, nor $\langle v_L \rangle = 0$. In this way, the model accepts v_o variations within the switching period and do not assumes volt-second balance on L.



$$\langle v_L \rangle = \langle v_{13} \rangle d + \langle v_{23} \rangle d_2 = L\frac{d\langle i_L \rangle}{dt}$$

$$\langle i_1 \rangle = \langle i_L \rangle \frac{d}{d + d_2}$$

$$\langle i_2 \rangle = \langle i_L \rangle \frac{d_2}{d + d_2}$$

$$\langle i_L \rangle = \langle i_1 \rangle + \langle i_2 \rangle$$

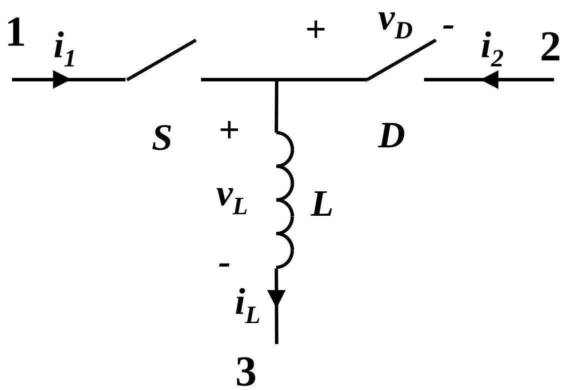
$$d_2 = -\frac{\langle v_{13} \rangle}{\langle v_{23} \rangle} d$$

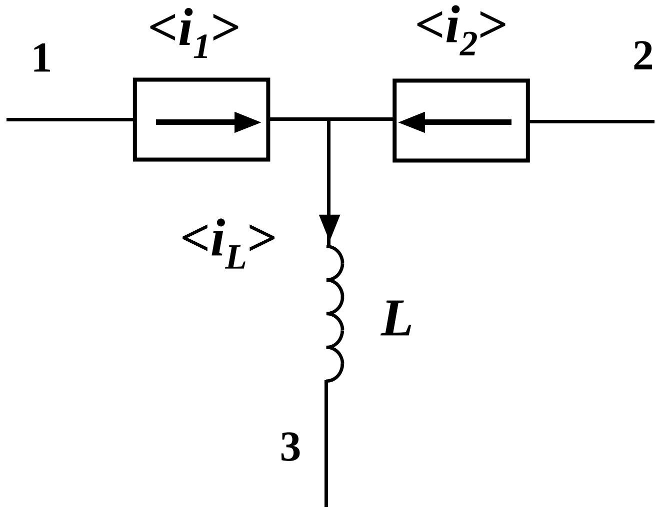
$$d_2 = \frac{2\langle i_L \rangle L}{\langle v_{13} \rangle dT} - d$$

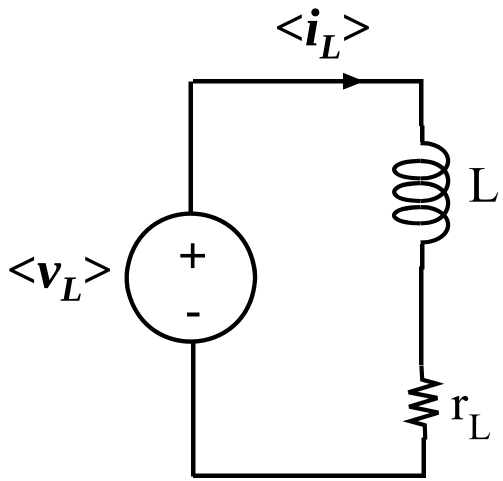
$\langle i_L \rangle$ is obtained by computing the integral of $\langle v_L \rangle$

Continuous dynamic model

Equivalent circuit



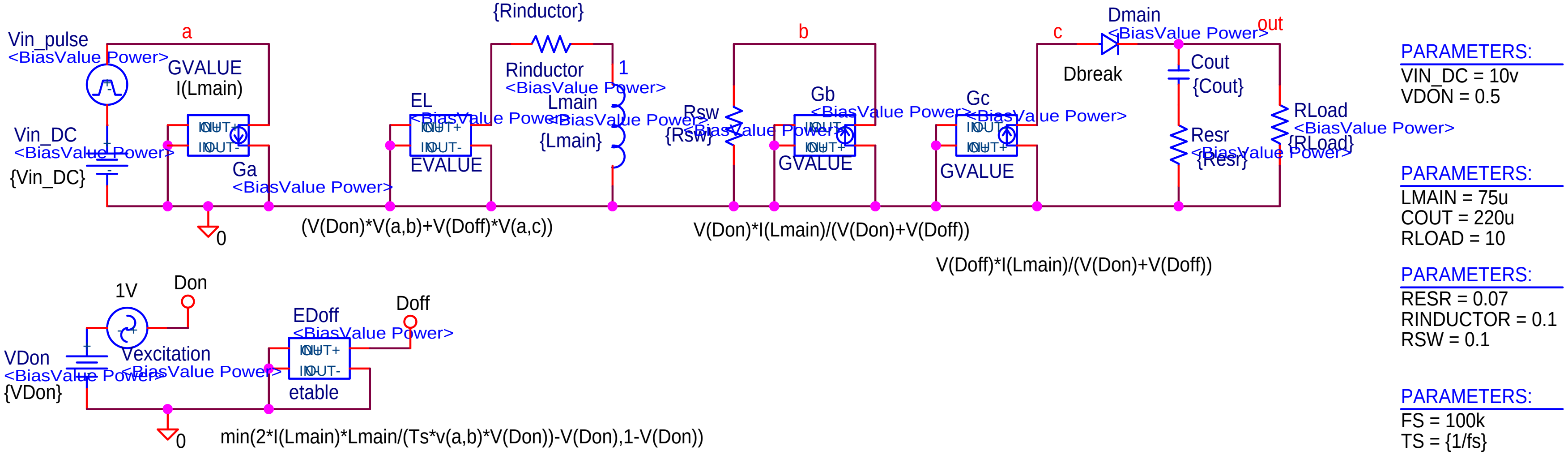




Continuous dynamic model. Simulation

SIM-Model under CCM & DCM for PWM Boost converter

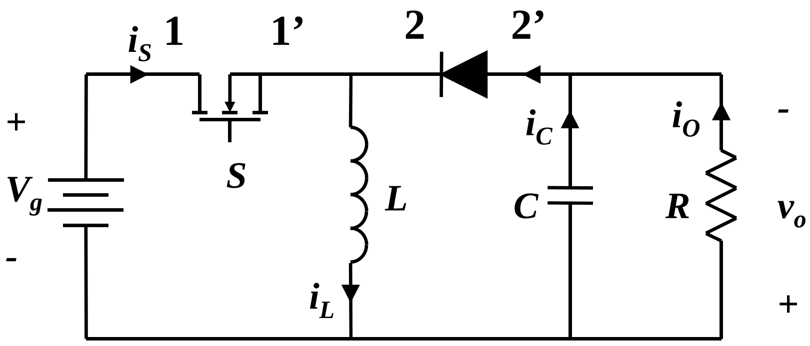
Boost.sch



Ref. 9:
S. Ben Yaakov "Computer aided design of power factor correction systems" Professional education seminars workbook. Vol. III. Seminar 11. APEC`03

Continuous dynamic model. Simulation

Similarly. Buck Boost example

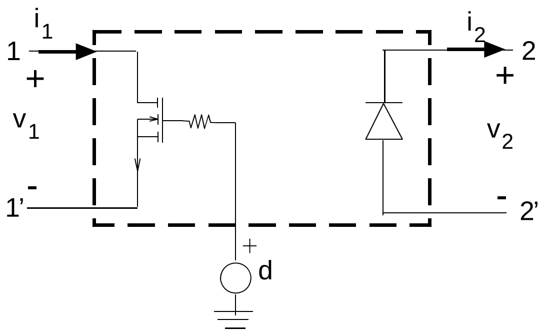


$$\langle v_1 \rangle = \left(\langle v_g \rangle + \langle v_o \rangle\right) d_2 + \langle v_g \rangle (1 - d - d_2)$$

$$\langle v_1 \rangle = \langle v_g \rangle (1 - d) + \langle v_o \rangle d_2$$

$$\langle v_2 \rangle = \left(\langle v_g \rangle + \langle v_o \rangle\right) d + \langle v_o \rangle (1 - d - d_2)$$

$$\langle v_2 \rangle = \langle v_g \rangle d + \langle v_o \rangle (1 - d_2)$$



$$\langle i_1 \rangle = \langle v_g \rangle \frac{d^2 T}{2L}$$

$$\langle i_2 \rangle = \langle v_g \rangle \frac{d d_2 T}{2L}$$

$$\langle i_L \rangle = \langle i_1 \rangle + \langle i_2 \rangle$$

The objective is to obtain d_2 without using <v_o>, nor <v_L>=0. In this way, the model accepts v_o variations within the switching period and do not assumes volt-second balance on L.

Continuous dynamic model. Simulation

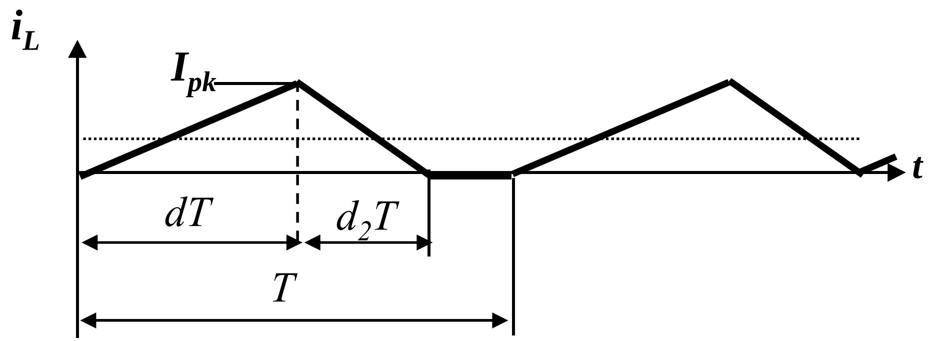
Buck Boost example

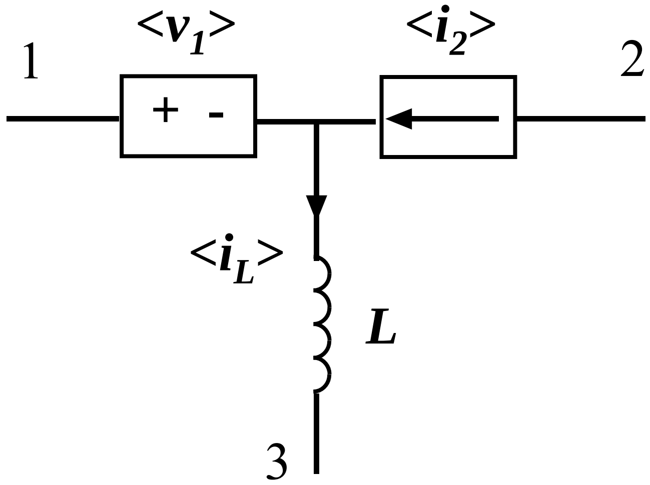
$$\langle i_L\rangle = \langle v_g\rangle \frac{d^2T}{2L} + \langle v_g\rangle \frac{dd_2T}{2L}$$

$$\langle i_L\rangle = \langle v_g\rangle \left(\frac{dT}{2L}\right)(d + d_2)$$

$$d_2 = \frac{\langle i_L\rangle}{\langle v_g\rangle}\left(\frac{2L}{dT}\right) - d \qquad R_e = \frac{2L}{d^2T}$$

$$d_2 = d\left(\frac{\langle i_L\rangle}{\langle v_g\rangle} R_e - 1\right)$$





The objective is to obtain d_2 without using $<v_o>$, nor $<v_L>=0$. In this way, the model accepts v_o variations within the switching period and do not assumes volt-second balance across L.

Buck Boost example. Reduced, complete or continuous model

ECEN 5807 Intro to converter sampled-data modeling - C.U. Boulder

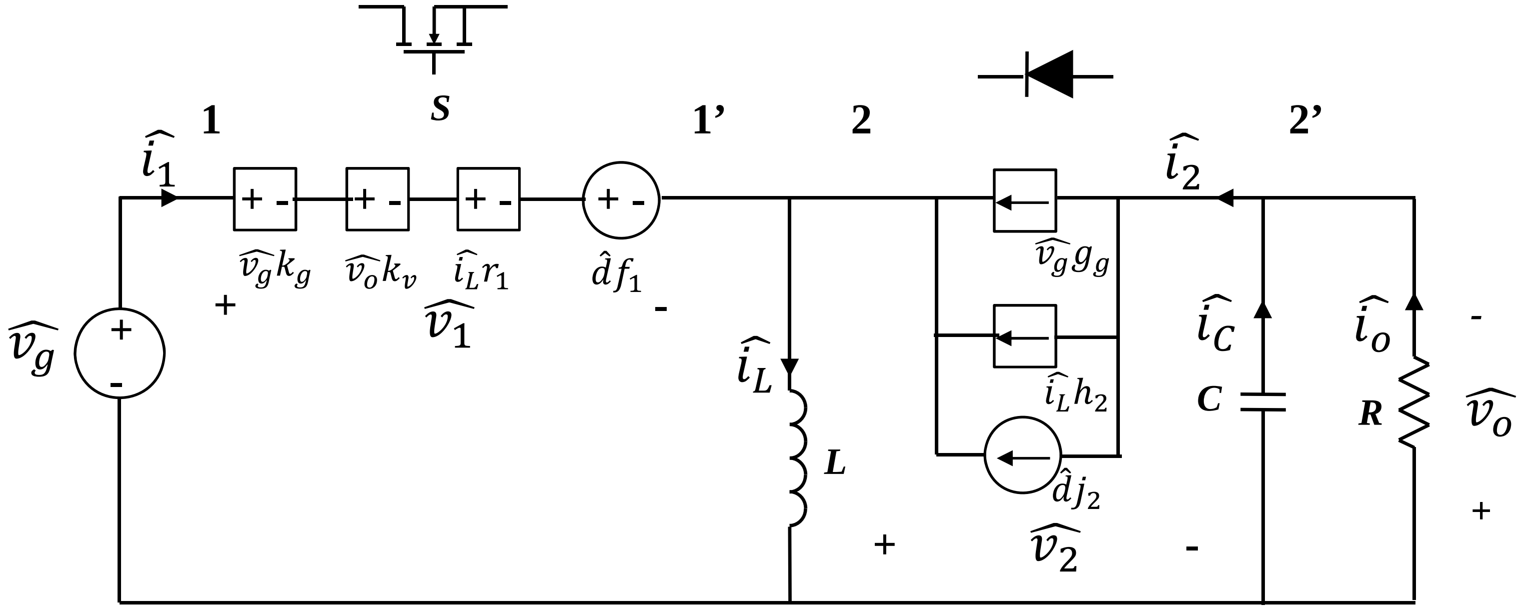
$$\langle v_1(t)\rangle_{T_s} = \gamma_1\left(\left\langle v_g(t)\right\rangle_{T_s}, \langle v_o(t)\rangle_{T_s}, \langle i_L(t)\rangle_{T_s}, d(t)\right)$$

$$\langle i_2(t)\rangle_{T_s} = \langle i_L(t)\rangle_{T_s} - \frac{\left\langle v_g(t)\right\rangle_{T_s}}{R_e} = \gamma_2\left(\left\langle v_g(t)\right\rangle_{T_s}, \langle i_L(t)\rangle_{T_s}, d(t)\right)$$

$$\widehat{v_1} = \widehat{v_g}k_g + \widehat{v_o}k_v + \widehat{i_L}r_1 + \hat{d}f_1$$

$$\widehat{i_2} = \widehat{v_g}g_g + \widehat{i_L}h_2 + \hat{d}j_2$$

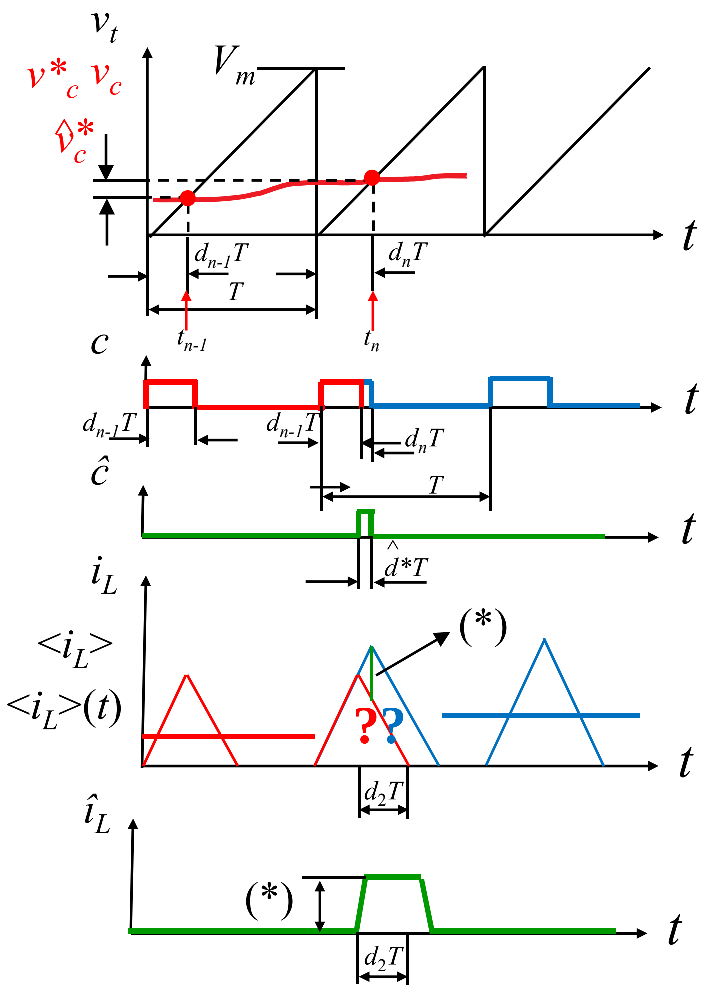
Buck Boost example. Reduced, complete or continuous model



Continuous model

ECEN 5807 Intro to converter sampled-data modeling - C.U. Boulder

Boost example



$$\widehat{v_c^*}(s) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \hat{v}_c(s - j\omega_s k)$$

$$\frac{v_c^*}{V_m} = d^* \qquad \omega_s = \frac{2\pi}{T}$$

$$\widehat{d^*}(s) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \hat{d}(s - j\omega_s k)$$

$$(*)\left(\frac{V_g}{L} - \frac{V_g - V_o}{L}\right)\widehat{d^*}(s)T = \frac{V_o}{L}\widehat{d^*}(s)T$$

$$\hat{i}_L(s) = \frac{V_o}{L}\widehat{d^*}(s)T\frac{1 - e^{-sD_2T}}{s}$$

$$\hat{i}_L(s) \cong \frac{V_o}{L}T\frac{1 - e^{-sD_2T}}{s}\frac{1}{T}\hat{d}(s)$$

$$\hat{i}_L(s) \cong \frac{V_o}{L}D_2T\frac{1 - e^{-sD_2T}}{sD_2T}\hat{d}(s) \qquad \frac{\hat{i}_L(s)}{\hat{d}(s)} \cong \frac{V_o}{L}D_2T\frac{1}{1 + \frac{s}{\omega_2}}$$

$$\frac{1 - e^{-sT}}{sT} \cong \frac{1}{1 + \frac{s}{\omega_p}} \qquad \omega_p = \frac{2}{T} = \frac{\omega_s}{\pi} \qquad \omega_2 = \frac{2}{D_2T}$$

Transient and small-signal simulation

Boost example, simulation verification

```
fs = 100 kHz
L = 1 mH
Vo = 400 V
Vg = 230 V
D = 0.2
```

PSpice models

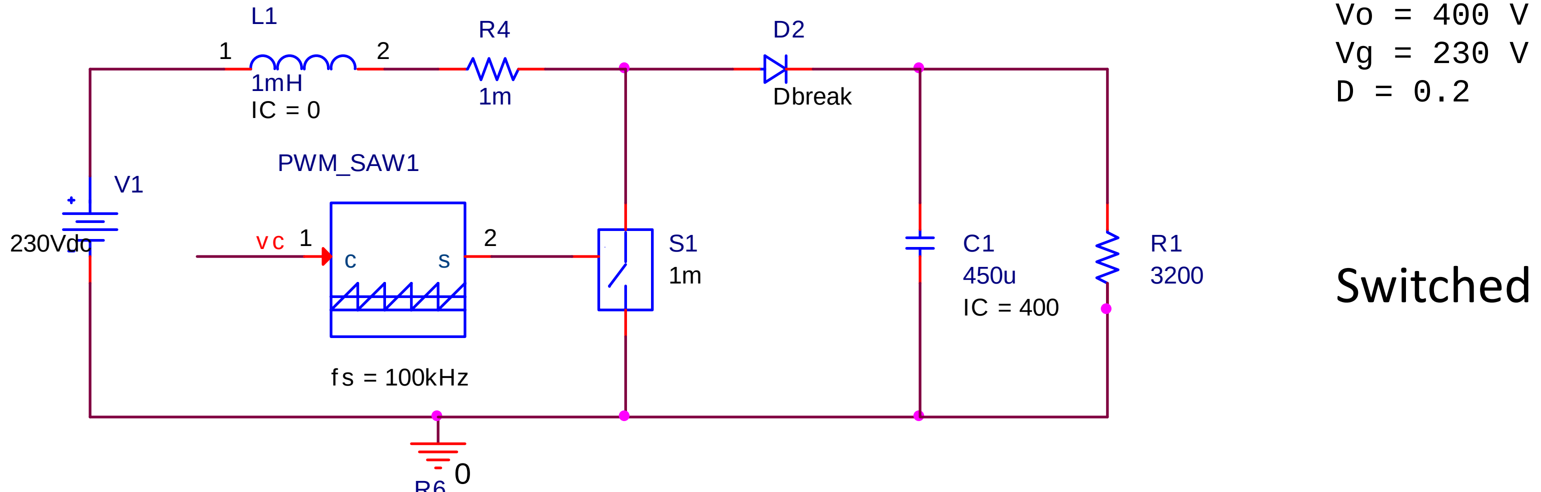
- Switched
- Reduced
- Complete
- Continuous

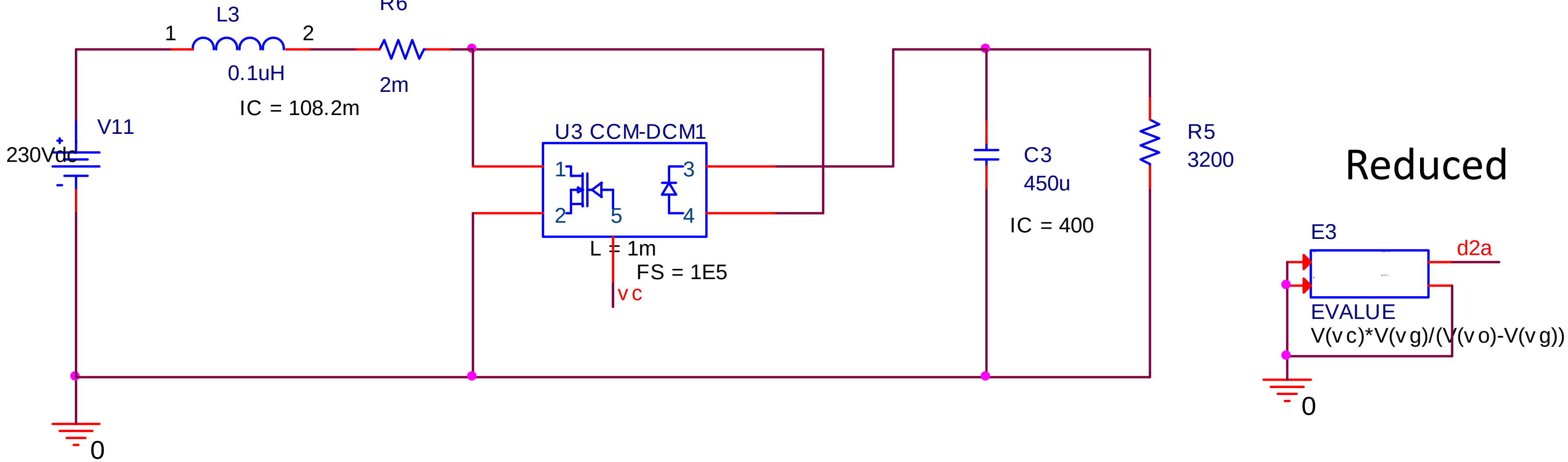
Computed discrete small-signal model

Dynamic models

Boost example, simulation verification

```
fs = 100 kHz
L = 1 mH
Vo = 400 V
Vg = 230 V
D = 0.2
```



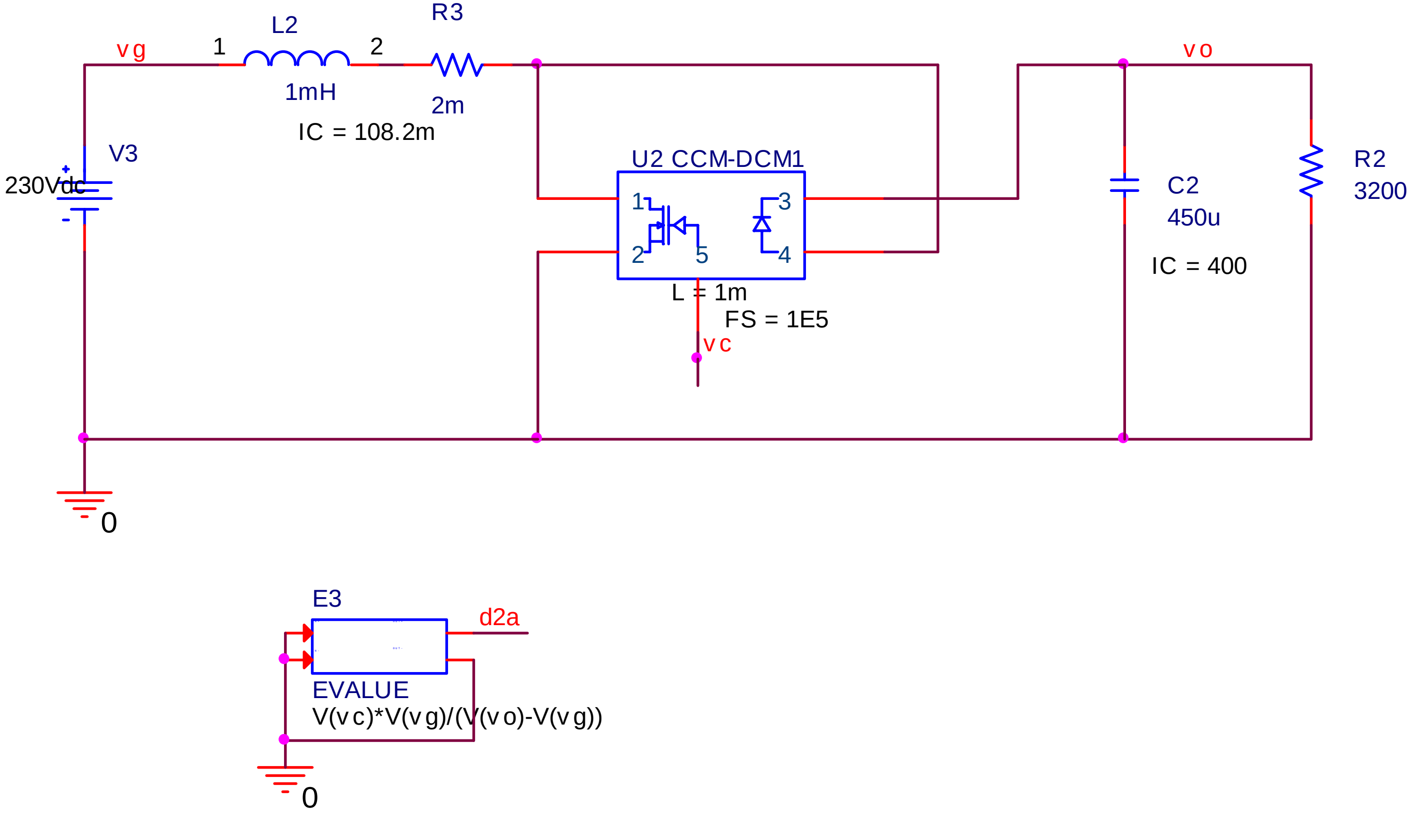


Dynamic models

Boost example, simulation verification

```
fs = 100 kHz
L = 1 mH
Vo = 400 V
Vg = 230 V
D = 0.2
```

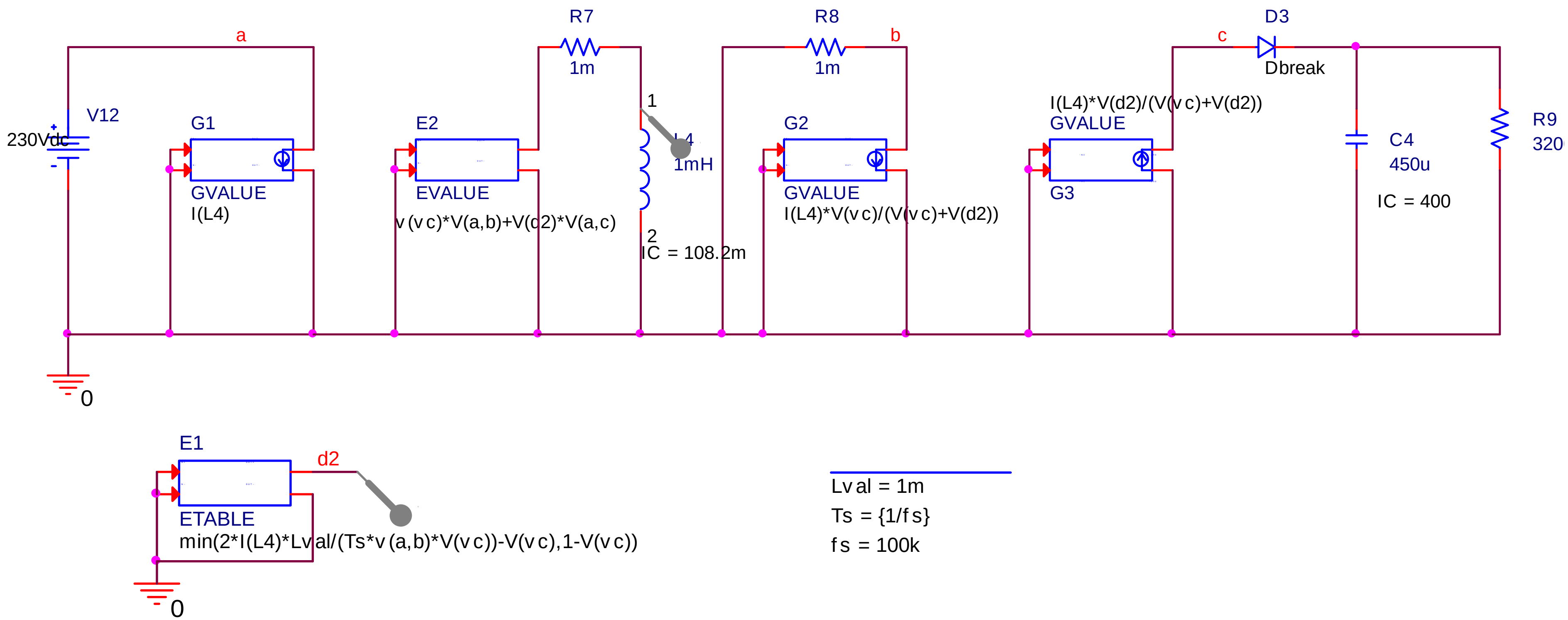
Complete



Dynamic models

Boost example, simulation verification

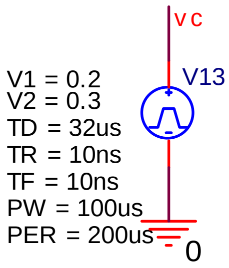
Continuous



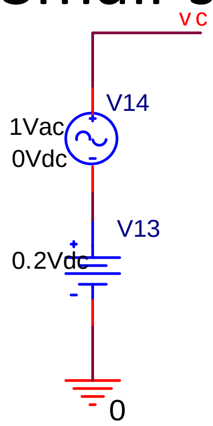
Boost example, simulation verification

fs = 100 kHz
L = 1 mH
Vo = 400 V
Vg = 230 V
D = 0.2

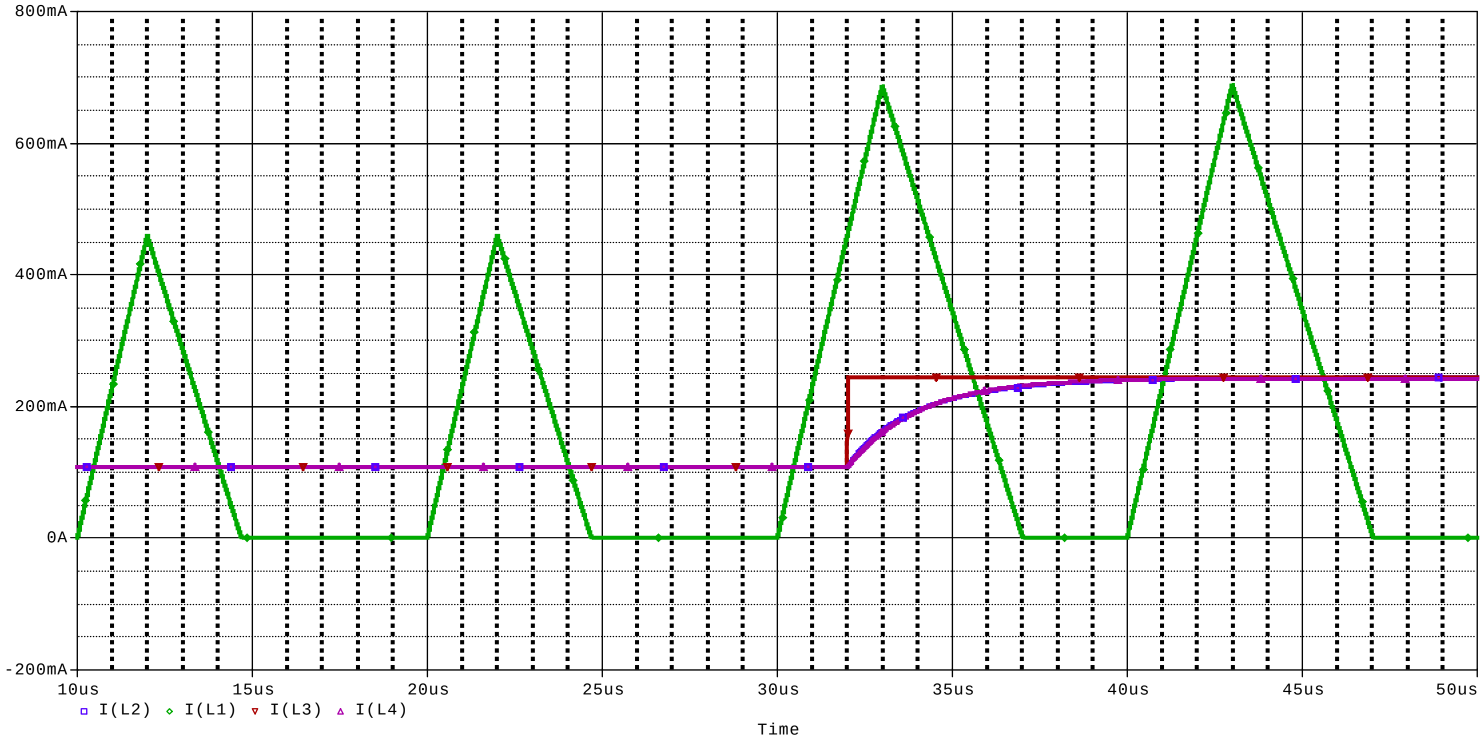
Transient (.tran)



Small-signal (.ac)



Transient simulation



PSpice models. Inductor current

Switched

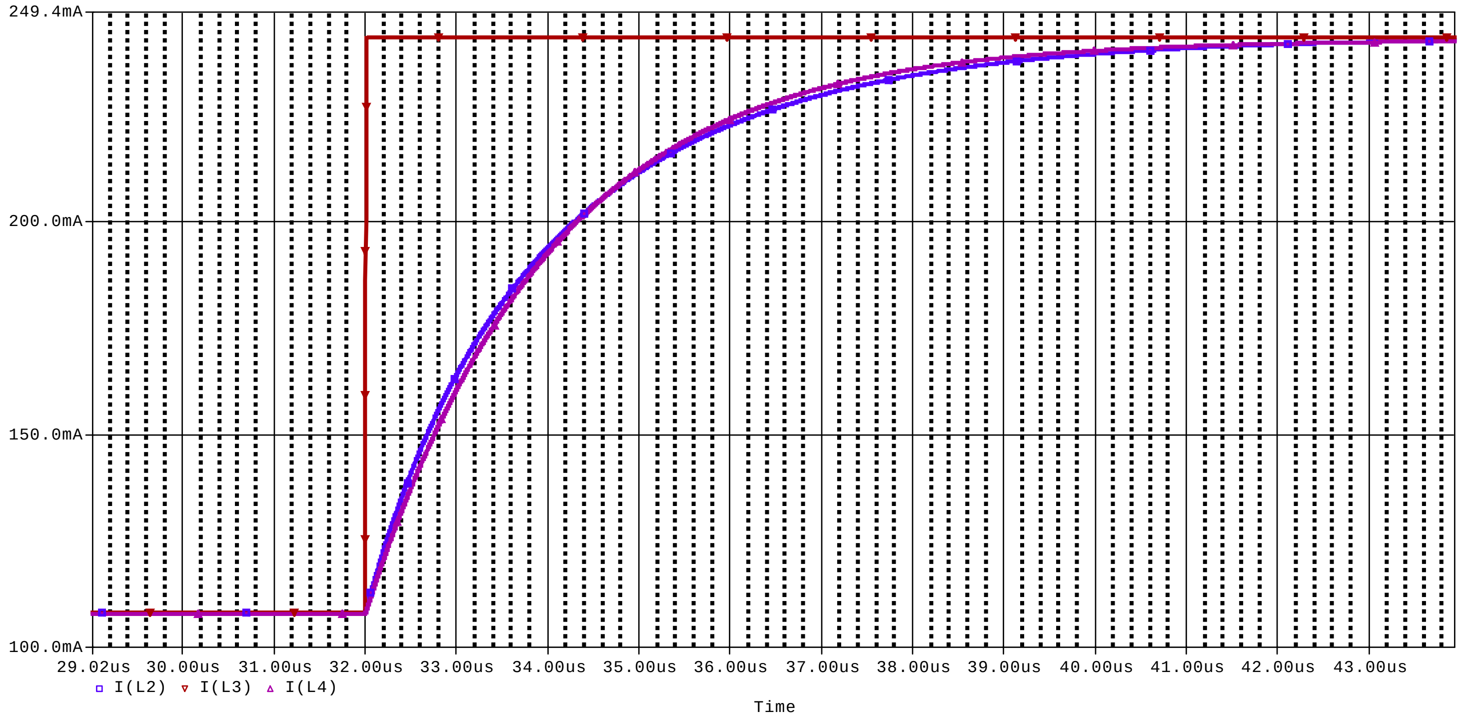
Reduced order

Complete

Continuous

Step D = 0.2 → D = 0.3

Transient simulation



PSpice models. Inductor current

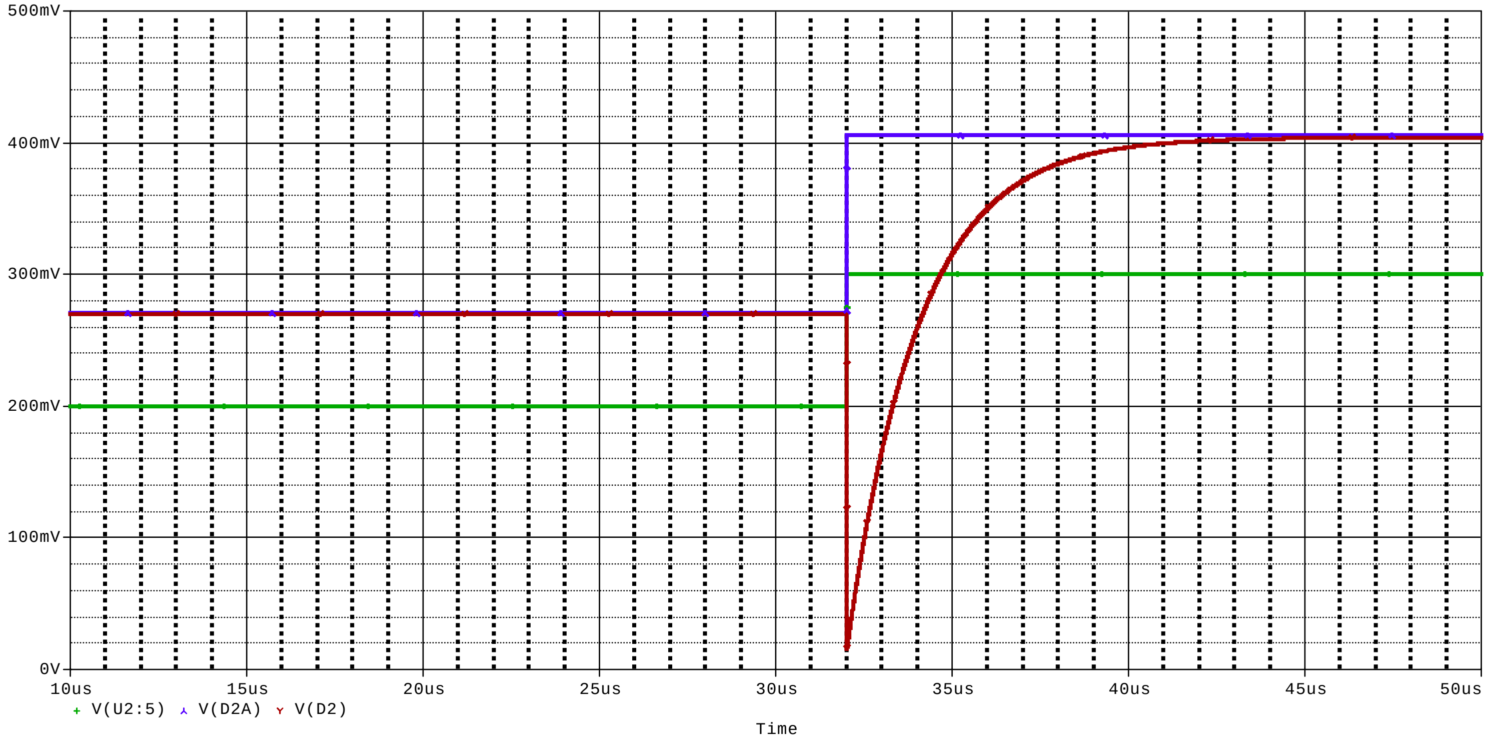
Reduced order

Complete

Continuous

Step D = 0.2 → D = 0.3

Transient simulation



PSpice models. d, d2

Step D = 0.2 → D = 0.3

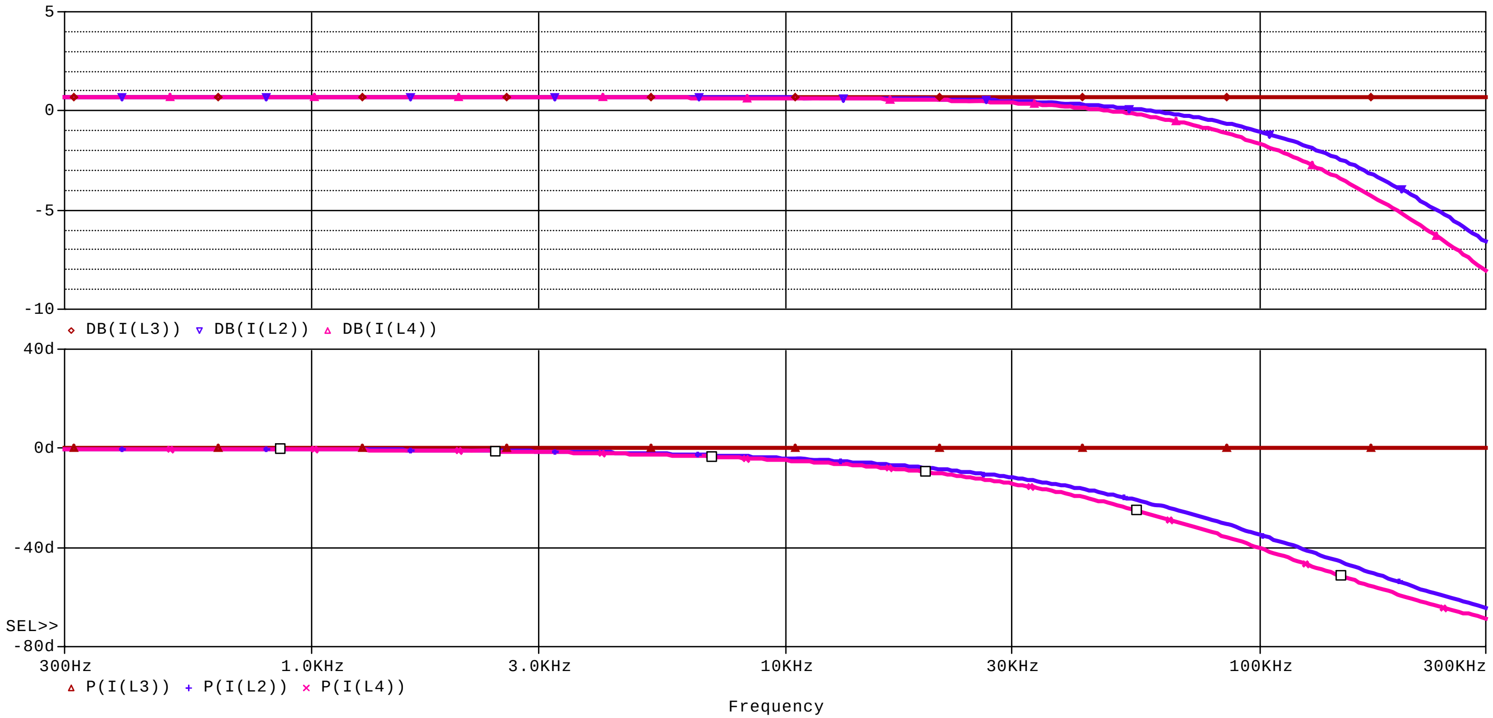
d

d2 Reduced, Complete

d2 Continuous

Reveals a RHP zero

Small-signal simulation



PSpice models G_{id}

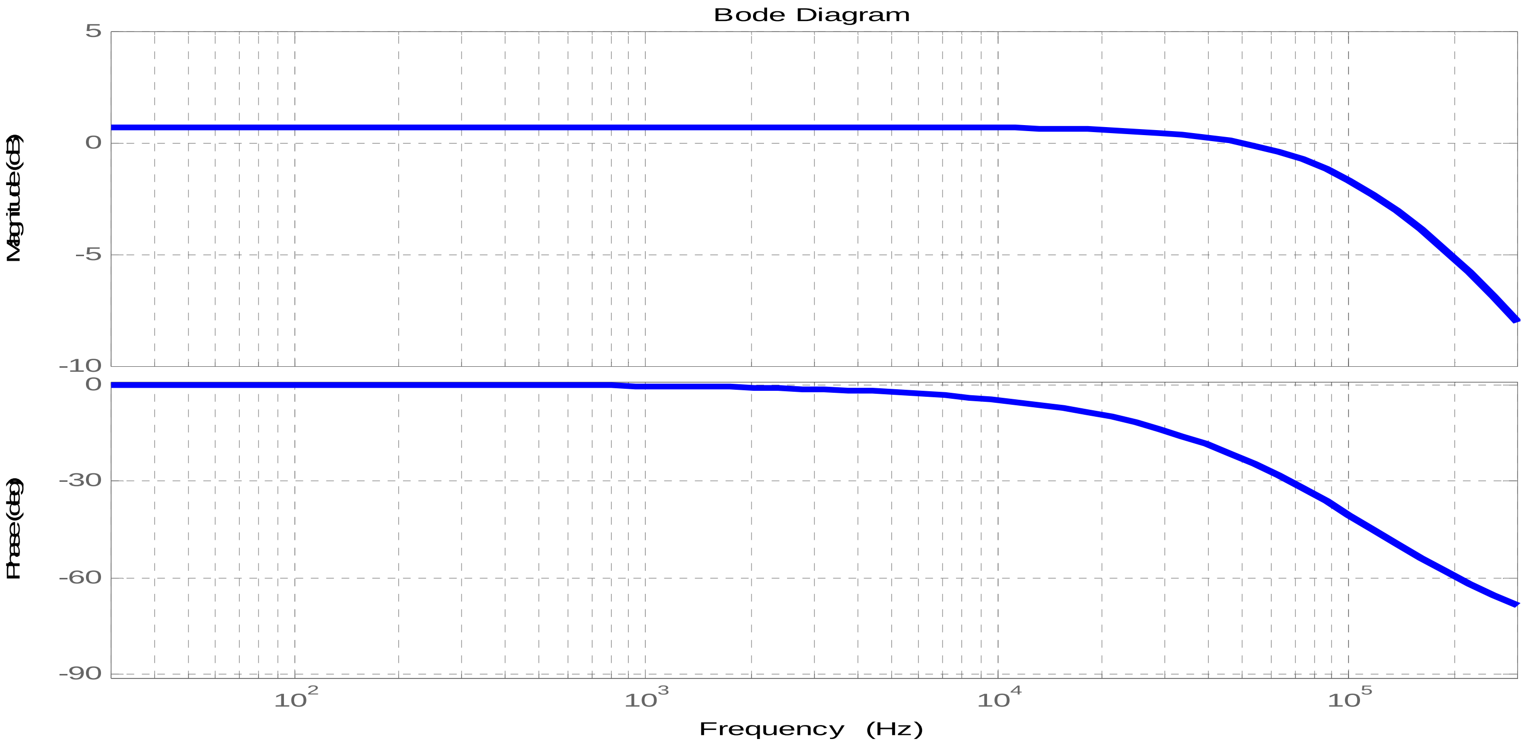
Reduced order

Complete

Continuous (*)

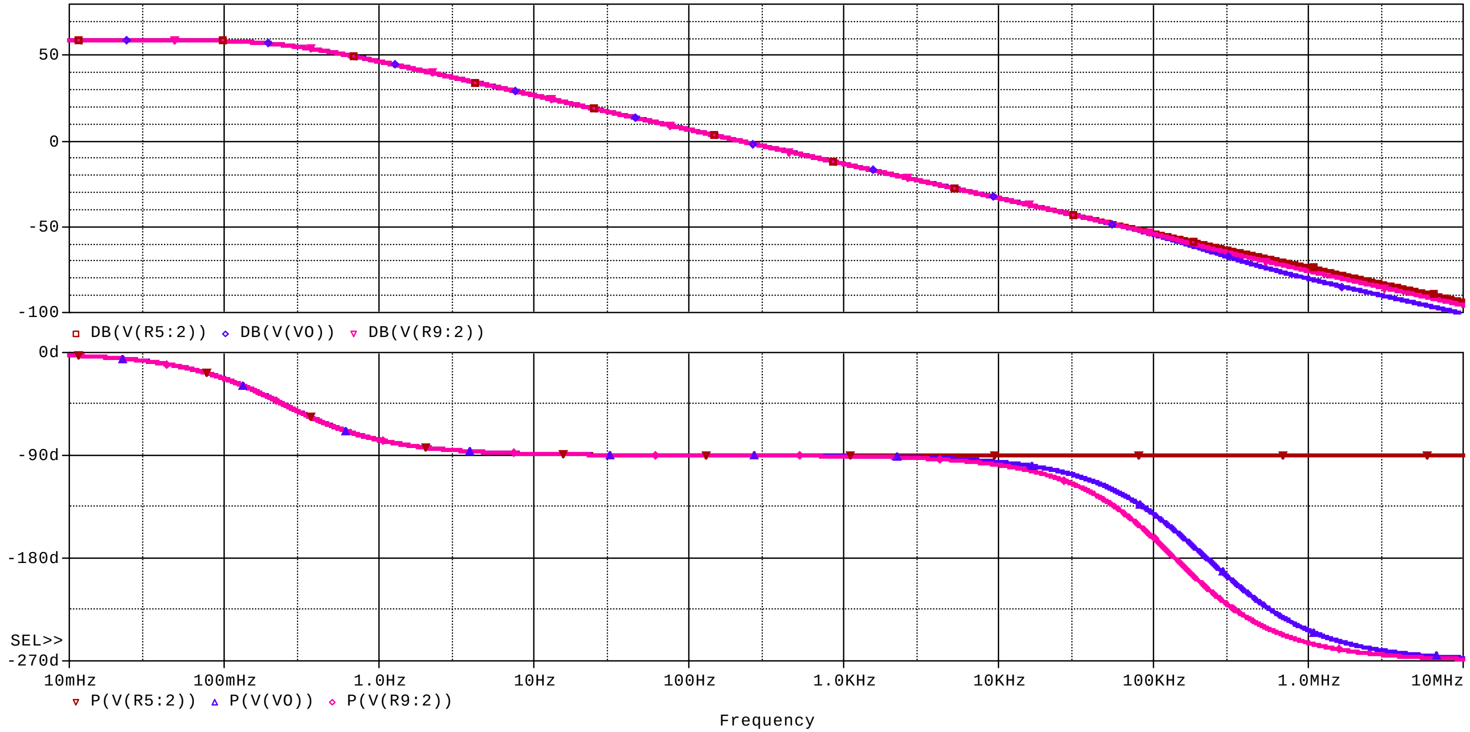
```
fs = 100 kHz
L = 1 mH
Vo = 400 V
Vg = 230 V
D = 0.2
```

```
clear;
clc;
fs = input(‘switching frequency [Hz] ');
L = input ('inductance [H]');
Vo = input (‘output voltage [V]');
Vg = input (‘input voltage [V] ');
D = input (‘duty cycle ');
Ts = 1/fs;
D2 = D*Vg/(Vo-Vg);
w2=2/(D2*Ts);
Go=Vo*D2*Ts/L;
s=tf('s');
Gid=Go*1/(1+s/w2);
H=bodeplot(Gid,{2*pi*30 , 2*pi*300000});
setoptions(H,'FreqUnits','Hz', 'PhaseMatching', 'on');
grid on
```



(*) Matches with G_{id} obtained with the continuous model

Small-signal simulation



PSpice models G_{vd}

Reduced order

Complete

Continuous

Conclusions and final remarks

CCM	***DCM (requires unidirectional switch)***
Lower current ripple	*Lower inductance, but larger size transformers and inductors*
Lower peak current	*Higher peak and RMS current*
RHPZ	*RHPZ goes above the switching frequency*
Double pole response. Current programmed control is often used.	*Single pole transfer function. Inductance influence goes to high frequency.*
No (Little) load dependent conversion ratio	*Load dependent conversion ratio*
Hard-switch	*ZCS, but ringing when i_L reaches zero due to parasitic L and C*
DC transformer equivalence	*Lossless resistor emulator equivalence*
Bidirectional power flow at low load reduces efficiency	*Not possible bidirectional power flow*

Conclusions and final remarks

1.- The operation in DCM requires a unipolar switch to prevent bidirectional power flow.

2.- Transformation expressions and transfer functions of converters in DCM differ from the converters in CCM.

3.- The output voltage of an ideal converter operating in DCM depends on the load.

4.- For the same D the V_o is higher in DCM than in CCM.

5.- Continuous model achieves a better description of the converter in DCM at high frequency

Conclusions and final remarks

6.- The complete order model models the switch network as a lossless resistor and a power source.

7.- DCM operation may be used to obtain PFC operation without specific current-mode control.

8.- In some systems it may have interest to design a power-mode control and operation in DCM may facilitate this operation mode.

9.- A converter model oriented to operate in both CCM and DCM should integrate both model plus a detection of the operation mode.

10.- Some converters are designed to operate in the critical model CCM-DCM to make the most of both operation modes. In this case switching frequency is different for different steady-state operation points.

11.- Some converters are designed by paralleling different stages operating in DCM or CCM-DCM to increase the power conversion rate. Interleaving of the current phase reduces the total current ripple and maintaining the dynamic performance of the conversion system equal to the dynamic performance of a single stage.

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